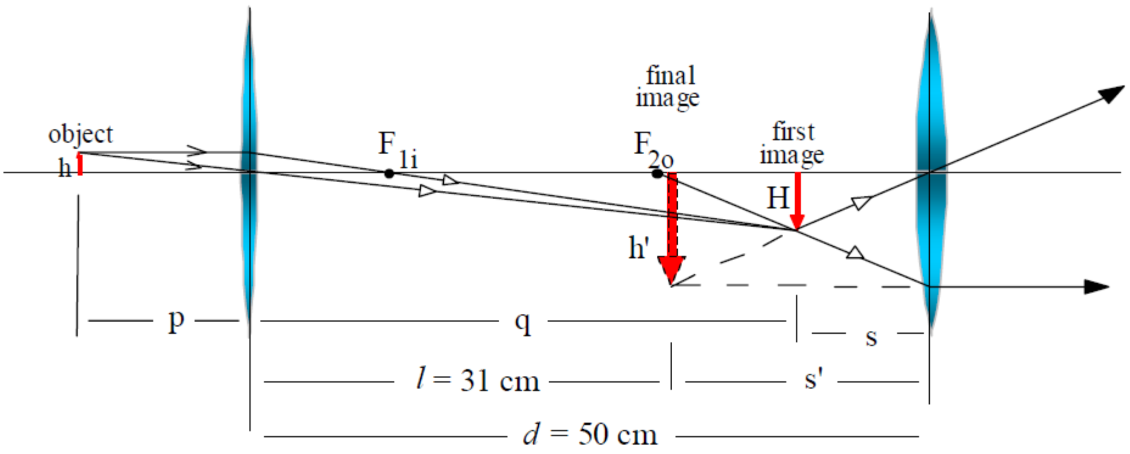
Answer on Question #52218-Physics-Optics

Two converging lenses having focal lengths of $f_1 = 10.0cm$ and $f_2 = 20.0cm$ are placed $d = 50.0cm$ apart. The final image is to be located between the lenses, at the position$x = 31.0cm$. (a) How far to the left of the first lens should the object be positioned? (b) What is the overall magnification to the system? (c) Is the final image upright or inverted?

Solution



A ray diagram helps us to imagine the situation. Since the final image (formed by the second lens) is not on the same side as the image rays (refracted be the second lens) the lens forms a virtual image. Tracing the chief ray and the image parallel ray (and the image focal ray) allows one to determine the position of the object for the second lens (image formed by the first lens). The found object (first image) is on the same side as the object rays therefore the object is real. Tracing the image chief, focal (and parallel) rays of the first lens allows one to identify the initial object.

The algebraic solution can be found from the lens equation applied to both lenses. Consistent with the figure

1) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1}$.

2) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_2}$.

Additionally

3) $q + s = d$.

4) $-s' + l = d$.

We obtained four equations with four unknown. The rest is algebra. We have to solve the above set for p. From the information given (eq. 4) we can determine the position of the final image (with respect to the second lens)

$$s = \left(\frac{1}{f_2} - \frac{1}{s'}\right)^{-1} = \left(\frac{1}{20cm} - \frac{1}{-19cm}\right)^{-1} \approx 9.74cm.$$

The position of the first image with respect to the first lens is therefore (from eq. 3)

$$q = d - s \approx 50cm - 9.74cm \approx 40.26cm.$$

Now from equation 1

$$p = \left(\frac{1}{f_2} - \frac{1}{q}\right)^{-1} = \left(\frac{1}{20cm} - \frac{1}{40.26cm}\right)^{-1} \approx 13.3cm.$$

(It makes sense to perform calculation step-by-step. We can check each step for possible discrepancies with the ray diagram.)

b) Considering similar triangles, formed by the optical axis, the chief rays, the objects and the images, one finds the following two proportionalities

$$\frac{-h'}{-H} = \frac{-s'}{s} \quad and \quad \frac{-H}{h} = \frac{q}{p}$$

From its definition, the magnification of the system is

$$M = \frac{h'}{h} = -\frac{-h}{-H} \cdot \frac{-H}{h} = \frac{s'}{s} \cdot \frac{q}{p} = \frac{-19.0cm}{9.74cm} \cdot \frac{40.3cm}{13.3cm} \approx -5.9.$$

c) Negative magnification, as well as the ray diagram, indicates that the final image is inverted.