Answer on Question #52205-Physics-Molecular Physics-Thermodynamics

a) Derive an equation of state $pV_\gamma =$ constant for an adiabatic process and show that an adiabat is steeper than an isotherm.

b) A block of copper whose expansivity, β, is $48.0 \cdot 10^{-6} k^{-1}$ and isothermal elasticity, ,E_T is $1.30 \cdot 10^{11} Nm^{-2}$is at atmospheric pressure and a temperature of 0°C. Its temperature is raised to 10°C. Calculate the final pressure when volume is kept constant. Express your answer in units of atmospheric pressure (atm).

c) Explain the working of a constant volume gas thermometer with the help of a neat and labeled diagram.

Solution

a) Let us use the first law of thermodynamics $\delta Q = C_V dT + PdV$ in order to derive an equation of state for adiabatic process. For an adiabatic process $Q = 0$, thus $C_V dT = -PdV$. Substituting the equation of ideal gas for one mole $P = \frac{RT}{V}$ into the right side of the previous equation, obtain $C_V dT = \frac{-RT}{V} dV$, or $\frac{dT}{T} = \frac{-R}{C_V}\frac{dV}{V}$.

Integrating from both sides, obtain $\frac{T_2}{T_1} = \frac{-R}{C_V} \ln \frac{V_2}{V_1}, \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{R}{C_V}}$ using the properties of logarithm. The last equation might be rewritten as $TV^{\frac{R}{C_V}} = const \; or \; TV^{\gamma - 1} = const$, where $\gamma = \frac{C_P}{C_V}$(Here we also used $C_P = C_V + R$).

Using $T = \frac{PV}{R}$ and last equation, obtain $PV^{\gamma-1+1} = PV^{\gamma} = const.$

The equation of isotherm is $P_V = const$, thus if $\gamma = \frac{C_P}{C_V} > 1$, the adiabat $P_V^{\gamma} = const$ is obviously.

b) For an isochoric process, we have

$$p_2 - p_1 = \beta E_T (T_2 - T_1).$$

On substituting the values of $\beta E_T (T_2 - T_1)$, we get

$$p_2 - p_1 = 48.0 \cdot 10^{-6} k^{-1} \cdot 1.30 \cdot 10^{11} Nm^{-2} (10K) = 624 \cdot 10^5 Nm^{-2} = 624 \; atm.$$

so that final pressure p_2 is

$$p_2 = (624 + 1) atm = 625 \; atm.$$

That is, to keep the volume of the copper block constant when its temperature is raised from 0°C to 10°C, one must increase the pressure to 625 atm.

c) The schematic diagram of a constant-volume gas thermometer is shown in Fig.1. The volume of an ideal gas in the sensing bulb D is kept constant by adjusting the level of mercury in the arm B of the manometer. The arm B and the arm A are connected by a flexible tube to form a U-tube manometer. The arm B is also connected to the gas bulb D via a capillary tube C, while the other arm A of the manometers is open to atmosphere and can be moved vertically to adjust the mercury level, so the mercury just touches the mark L of the capillary. The pressure in the bulb b is used as a thermometric property and can be given by

$$\mathrm{p} = \mathrm{p_{atm}} + \rho \, \mathrm{gh} \; (1)$$

where $\mathrm{p_{atm}}$ is atmospheric pressure; ρ is the density of the mercury; h is the mercury column in manometer.

The gas bulb D is first placed in constant-temperature bath at the triple point temperature T_{tp} of water and the level of mercury is adjusted to touch the mark L by moving the manometer arm A up and down.

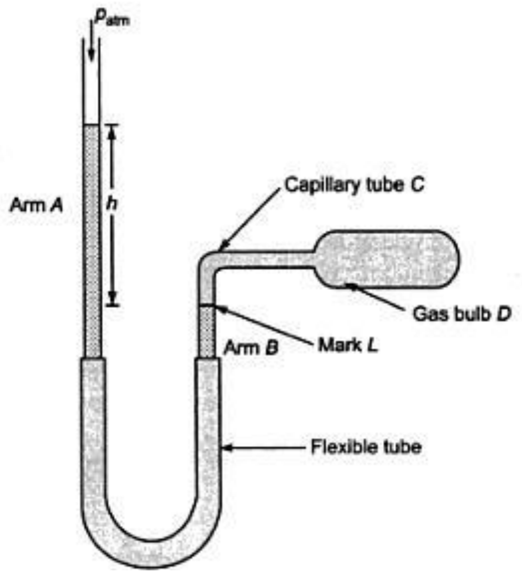

Fig.1.

As the volume of the bulb becomes constant and the height difference of the mercury in the two arms is recorded as h_{tp} the pressure, p_{tp} corresponding to the mercury column at the triple point is calculated by Eq. (1)

Now the bulb is brought in contact with a system whose temperature T, is to be measured. Again, in a similar manner, by keeping the volume of gas in the bulb constant, the height difference of the mercury in the two arms is recorded and the corresponding new pressure p is calculated by Eq. (2).

From the ideal gas equation, the new temperature is given by

$$\mathrm{T} = 273.15\mathrm{K}\,\frac{p}{p_{tp}}.$$

where 273.15K is the triple point temperature of water.