Answer on Question #52141-Physics-Molecular Physics | Thermodynamics

1/A cylinder containing n mol of an ideal gas undergoes an adiabatic process.

(a) Show that the work done on the gas is $W = \left(\frac{1}{\gamma - 1}\right)\left(P_f V_f - P_i V_i\right)$

(b) Starting with the first law of thermodynamics in differential form, prove that the work done on the gas is also equal to $nC_V\left(T_f - T_i\right)$. Show that this result is consistent with the equation in part (a).

Solution

(a) The differential work done on the system by the surroundings is

$$\delta w = -pdV$$

The adiabatic condition for an ideal gas is $pV^{\gamma} = K$, where K is a constant. Note that this also implies that $p_{initial}(V_{initial})^{\gamma} = p_{final}\left(V_{final}\right)^{\gamma} = K$.

Rearranging this, we have that $p = K\,V^{-\gamma}$.

Plugging this into the expression for the work:

$$\delta w = -KV^{-\gamma}dV$$

Now just integrate both sides of the equation:

$$\Delta w = -\left(\frac{K}{1-\gamma}\right)\left[\left(V_{final}\right)^{1-\gamma} - (V_{initial})^{1-\gamma}\right].$$

Remember from above that $K = p_{initial}(V_{initial})^{\gamma} = p_{final}\left(V_{final}\right)^{\gamma}$, so

$$\Delta w = \left(\frac{1}{\gamma-1}\right)\left[p_{final}\left(V_{final}\right)^{\gamma}\left(V_{final}\right)^{1-\gamma} - p_{initial}(V_{initial})^{\gamma}(V_{initial})^{1-\gamma}\right]$$

$$\Delta w = \left(\frac{1}{\gamma-1}\right)\left[p_{final}V_{final} - p_{initial}V_{initial}\right].$$

This is what we are supposed to show.

(b) Let's start with the differential form of the first Law:

$$dE = \delta q + \delta w$$

For an adiabatic process, $\delta q = 0$, so:

$$dE = \delta w$$

$$\Delta E = \Delta w.$$

So the change in internal energy is equal to the work done on the system by the surroundings, which is what we calculated above.

The internal energy of an ideal gas depends only on temperature, and from the definition of the constant-volume molar heat capacity:

$$\left(\frac{\partial E}{\partial T}\right)_V = nC_v.$$

This implies that:

$$dE = nC_v\,dT.$$

If we assume C_v is a constant (which it is for an ideal gas), this is easily integrated to obtain:

$$\Delta E = nC_v\Delta T, where\ \Delta T = T_{final} - T_{initial}.$$

Above, we showed that for an adiabatic process, $\Delta E = \Delta w$, so

$$\Delta w = nC_v\Delta T.$$

The state equation for ideal gas is

$$pV = nRT.$$

Thus

$$\left[p_{final}V_{final} - p_{initial}V_{initial}\right] = nR\left(T_{final} - T_{initial}\right).$$

And

$$C_V = \frac{R}{\gamma - 1}.$$

Thus, this result is consistent with the equation in part (a).

2/ A vertical cylindrical tank contains 1.80 mol of an ideal gas under a pressure of 1.00 atm at 20.0oC. The round part of the tank has a radius of 10.0 cm, and the gas is supporting a piston that can move up and down in the cylinder without friction.

(a) What is the mass of this piston?

(b) How tall is the column of gas that is supporting the piston?

Solution

(a) The pressure is

$$p = \frac{F}{A} = \frac{mg}{A}.$$

the mass of this piston is

$$m = \frac{pA}{g} = \frac{1.01 \cdot 10^5 Pa \cdot \pi(0.1\ m)^2}{9.8\frac{m}{s^2}} = 324\ kg.$$

(b) From the state equation of ideal gas

$$pV = nRT \to V = \frac{nRT}{p}.$$

The height of the column is

$$h = \frac{V}{A} = \frac{nRT}{pA} = \frac{1.80\ mol \cdot 8.31 \frac{J}{molK} \cdot 293.15K}{1.01 \cdot 10^5 Pa \cdot \pi (0.1\ m)^2} = 1.38\ m.$$

Answer: (a) $\mathbf{324\ kg}$; (b) $\mathbf{1.38\ m}$.