Answer on Question #52140-Physics-Mechanics | Kinematics | Dynamics

1/ (10 pts) A hollow sphere with an inner radius R and outer radius 2R is made of material of density D and is floating in a liquid of density 2D. The interior is now filled with material of density D' so that the sphere just floats completely submerged. Find D' in terms of D.

Solution

Per Archimedes, the force on an object is equal to the weight of displaced fluid

$$DV_{hollow\ sphere} = 2D \cdot V_{liquid}.$$

$$\frac{4}{3}\pi((2R)^3 - R^3) = 2V_{liquid} \to V_{liquid} = \frac{14}{3}\pi R^3 = \frac{1}{2}V_{hollow\ sphere}.$$

Now

$$DV_{hollow\ sphere} + D' \cdot \frac{4}{3}\pi R^3 = 2D\frac{4}{3}\pi(2R)^3.$$

$$D' \cdot \frac{4}{3}\pi R^3 = 2D\frac{4}{3}\pi(2R)^3 - D\frac{14}{3}\pi R^3.$$

$$D' = D\frac{64 - 14}{4} = 12.5D.$$

Answer: $12.5D$.

2/ (30 pts) A column of mercury of 10 cm length is contained in the middle of a narrow horizontal 1 m long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance will the column of mercury be displaced if the tube is held vertically?

Solution

Let the column of Hg will come down by a distance x. And at this point the air pressure at upper part of the tube is P_1 and the lower part of the tube is P_2. The cross-section of the tube is a. So,

$$P_1 + 10\ cm\ of\ Hg = P_2$$

Now using equation of state on the upper part of the tube

$$76 \cdot 45 \cdot a = P_1 \cdot (45 + x) \cdot a \to P_1 = \frac{76 \cdot 45}{45 + x} cm\ of\ Hg$$

Same for the lower part of the tube,

$$76 \cdot 45 \cdot a = P_2 \cdot (45 - x) \cdot a \to P_2 = \frac{76 \cdot 45}{45 - x} cm\ of\ Hg$$

Therefore,

$$\frac{76 \cdot 45}{45 + x} + 10 = \frac{76 \cdot 45}{45 - x} \to \frac{76 \cdot 45}{45 - x} - \frac{76 \cdot 45}{45 + x} = 10 \to \frac{x}{45^2 - x^2} = \frac{5}{76 \cdot 45}$$

$$x^2 + 684x - 45^2 = 0$$

$$x = \frac{-684 \pm \sqrt{684^2 + 4 \cdot 45^2}}{2} = 2.95\ cm.$$

Answer: $2.95\ cm$.