Answer on Question #51972 – Physics – Other

Question.

A 2000 kg satellite orbits the earth at a height of 300 km. What is the speed of the satellite and its period? TakeG=6.67×10−11Nm2/kg2, Mass of the earth is 5.98×1024kg.

Given:

$m = 2000\ kg$

$h = 300\ km = 3 \cdot 10^5\ m$

$G = 6.67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$

$M = 5.98 \cdot 10^{24} kg$

Find:

$v = ?$

$T = ?$

Solution.

If the satellite moves in circular motion, then the net centripetal force acting upon this orbiting satellite is given by the relationship:

$$F_{net} = \frac{mv^2}{r}$$

This net centripetal force is the result of the gravitational force that attracts the satellite towards the central body and can be represented as:

$$F_{grav} = G\frac{mM}{r^2}$$

The above expressions for centripetal force and gravitational force can be set equal to each other. Thus,

$$\frac{mv^2}{r} = G\frac{mM}{r^2}$$

Therefore,

$$v = \sqrt{\frac{GM}{r}}$$

In our case, $r = R + h$, where $R = 6.37 \cdot 10^6\ m$. That's why,

$$v = \sqrt{\frac{GM}{R + h}}$$

By definition

$$T = \frac{2\pi}{\omega}$$

And

$$\omega = \frac{v}{r} = \frac{\sqrt{GM}}{(R+h)^{\frac{3}{2}}}$$

So,

$$T = \frac{2\pi(R+h)^{\frac{3}{2}}}{\sqrt{GM}}$$

Calculate:

$$v = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24}}{6.67 \cdot 10^{6}}} = 7730 \ \frac{m}{s}$$

$$T = \frac{2\pi(R+h)^{\frac{3}{2}}}{\sqrt{GM}} = \frac{6.28 \cdot (6.67 \cdot 10^{6})^{\frac{3}{2}}}{\sqrt{6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24}}} = 5420 \ s$$

Answer.

$$v = \sqrt{\frac{GM}{R+h}} = 7730 \ \frac{m}{s}$$

$$T = \frac{2\pi(R+h)^{\frac{3}{2}}}{\sqrt{GM}} = 5420 \ s$$