Answer on Question #51961 - Physics - Other

The resultant of vectors $\vec{A}$ and $\vec{B}$ has a magnitude of 20 units. $\vec{A}$ has a magnitude of 8 units, and the angle between $\vec{A}$ and $\vec{B}$ is $\varphi = 40°$. Calculate the magnitude of $\vec{B}$.

Solution:

Since $\left(\vec{A}+\vec{B}\right)^2 = \left|\vec{A}+\vec{B}\right|^2$, we obtain

$$\left|\vec{A}\right|^2 + 2\left|\vec{A}\right|\cdot\left|\vec{B}\right|\cos\varphi + \left|\vec{B}\right|^2 = \left|\vec{A}+\vec{B}\right|^2$$

Or equivalently

$$\left|\vec{A}\right|^2\cos^2\varphi + 2\left|\vec{A}\right|\cdot\left|\vec{B}\right|\cos\varphi + \left|\vec{B}\right|^2 = \left|\vec{A}+\vec{B}\right|^2 - \left|\vec{A}\right|^2\sin^2\varphi$$

This equation has only one positive root:

$$\left|\vec{B}\right| = \sqrt{\left|\vec{A}+\vec{B}\right|^2 - \left|\vec{A}\right|^2\sin^2\varphi} - \left|\vec{A}\right|\cos\varphi$$

Since $\left|\vec{A}+\vec{B}\right| = 20$, $\left|\vec{A}\right| = 8$ and $\varphi = 40°$, we obtain

$$\left|\vec{B}\right| = \sqrt{20^2 - 8^2\sin^2 40°} - 8\cos 40° = 13.2$$

Answer: $\left|\vec{B}\right| = \sqrt{\left|\vec{A}+\vec{B}\right|^2 - \left|\vec{A}\right|^2\sin^2\varphi} - \left|\vec{A}\right|\cos\varphi = 13.2.$