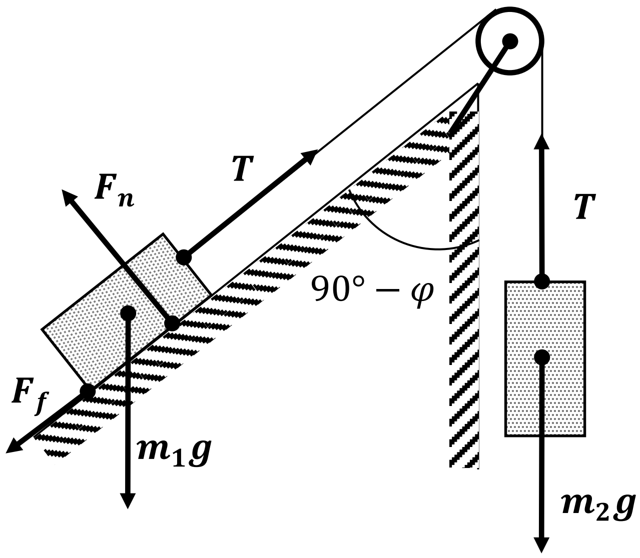
Answer on Question#51955 - Physics - Other

A $m_1 = 15$ kg block rests on the surface of a plane inclined at an of $\varphi = 30°$ to the horizontal. A light inextensible string passing over a small, smooth pulley at the top of the plane connects the block to another $m_2 = 13$ kg block hanging freely. The coefficient of kinetic friction between the 15 kg block and the plane is $\mu = 0.25$. Find the acceleration a of the blocks.

Solution:



The force of friction is acting downward, since the projection of weight of the block 1 on the inclined plane is smaller than the weight of block 2:

$$m_1 g \sin\varphi = 15\text{kg} \cdot 10\frac{\text{m}}{\text{s}^2} \cdot \sin 30° = 75\text{N} < 130\text{N} = 13\text{kg} \cdot 10\frac{\text{m}}{\text{s}^2} = \text{m}_2 g$$

The normal reaction F_n is given by

$$F_n = m_1 g \cos\varphi$$

The force of friction F_f is given by

$$F_f = F_n \cdot \mu = m_1 g \mu \cos\varphi$$

Let's first apply Newton's second law to mass m_1 (projection on the inclined plane, upward direction is positive)

$$m_1 a = T - F_f - m_1 g \sin\varphi,$$

where T – is the tension force of the string.

Let's now apply Newton's second law to mass m_2 (projection on the vertical axis, downward direction is positive)

$$m_2 a = m_2 g - T$$

Summing the last two equations we obtain

$$(m_1 + m_2)a = m_2 g - F_f - m_1 g \sin\varphi$$

Therefore the acceleration a of the blocks is (we put $g = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$)

$$a = \frac{m_2 g - F_f - m_1 g \sin\varphi}{m_1 + m_2} = \frac{m_2 g - m_1 g\mu \cos\varphi - m_1 g \sin\varphi}{m_1 + m_2} =$$

$$= \frac{13\mathrm{kg} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} - 15\mathrm{kg} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 0.25 \cdot \cos 30° - 15\mathrm{kg} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \sin 30°}{15\mathrm{kg} + 13\mathrm{kg}} = 0.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

<u>Answer:</u> $a = \frac{m_2 g - m_1 g\mu \cos\varphi - m_1 g \sin\varphi}{m_1 + m_2} = 0.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$.