Answer on Question #51827, Physics, Mechanics | Kinematics | Dynamics

Which of the following is NOT correct?

$\vec{k} \cdot \vec{k} = 1$

$\vec{i} \times \vec{j} = \vec{k}$

$\vec{i} \times \vec{k} = -\vec{j}$

$\vec{j} \cdot \vec{j} = 0$

Solution:

Unit vectors may be used to represent the axes of a Cartesian coordinate system. For instance, the unit vectors in the direction of the x, y, and z axes of a three dimensional Cartesian coordinate system are

$$\hat{\mathbf{i}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \hat{\mathbf{j}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \hat{\mathbf{k}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We first calculate that the dot product of the unit vector $\vec{i}$ with itself is unity

$$\vec{i} \cdot \vec{i} = |\vec{i}||\vec{i}| \cos(0) = 1$$

since the unit vector has magnitude $|\vec{i}|$=1 and cos(0) =1. We note that the same rule applies for the unit vectors in the y and z directions:

$$\vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$$

The cross product is

$$\boldsymbol{a} \times \boldsymbol{b} = |a||b| \sin t \, \boldsymbol{n}$$

where n is a unit vector perpendicular to the plane in which a and b lie.

Since sin 0 = 0 and sin 90 = 1 and each vector is of unit length, we have

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0, \text{(the zero vector)}.$$

Also, $\vec{i} \times \vec{j} = \vec{k}$ and $\vec{j} \times \vec{k} = \vec{i}$ and $\vec{k} \times \vec{i} = \vec{j}$

while $\vec{j} \times \vec{i} = -\vec{k}$ and $\vec{k} \times \vec{j} = -\vec{i}$ and $\vec{i} \times \vec{k} = -\vec{j}$.

Answer: $\vec{j} \cdot \vec{j} = 0$ is NOT correct.