Answer on Question #51673, Physics, Mechanics | Kinematics | Dynamics

Assume that a tunnel is dug across the earth (radius R) passing through its center. Find the time a particle takes to cover the length of the tunnel if

a) it is projected into the tunnel with a velocity (gR)^1/2

b) it is released from a height R above the tunnel

c) it is thrown vertically upward along the length of the tunnel with a speed (gR)^1/2

Solution:

(a) Let T be the time period of the oscillatory motion of the particle, x_t ,v_t and a_t the displacement from the center, velocity and acceleration at time t. Then

$$a_t = \frac{F}{m} = GM\frac{x_t^3}{R^3 x_t^2} = GMx_t/R^3$$

The period of oscillation

$$T = 2\pi\left(\frac{x_t}{a_t}\right)^{\frac{1}{2}} = 2\pi\left(\frac{R^3}{GM}\right)^{\frac{1}{2}} = 2\pi\left(\frac{R}{g}\right)^{\frac{1}{2}}$$

The angular velocity

$$\omega = \left(\frac{g}{R}\right)^{\frac{1}{2}}$$

If A be the amplitude of the motion then

$$v_t^2 = \omega^2(A^2 - R^2) = \frac{g(A^2 - R^2)}{R}$$

But at the surface of the Earth we have $v_t = (gR)^{\frac{1}{2}}$

$$gR = \frac{g(A^2 - R^2)}{R}$$

$$R^2 = A^2 - R^2$$

$$A = \sqrt{2}R$$

Let t=t_1 and t=t_2 be two successive instants when value of x_t changes from +R to –R and therefore $t_2 - t_1$is the time taken to cover the length o the tunnel. But

$$R = \sqrt{2}R\sin\omega t_1$$

$$\sin\omega t_1 = \frac{1}{\sqrt{2}}$$

$$\omega t_1 = \frac{3\pi}{4}$$

and

$$-R = \sqrt{2}R\sin\omega t_2$$

$$\sin\omega t_2 = -\frac{1}{\sqrt{2}}$$

$$\omega t_2 = \frac{5\pi}{4}$$

$$\omega t_2 - \omega t_1 = \frac{\pi}{2}$$

$$t_2 - t_1 = \frac{\pi}{2\omega} = \frac{\pi}{2}\left(\frac{R}{g}\right)^{\frac{1}{2}}$$

b) In this case the particle is released from height R above the tunnel being a case of free fall motion till the particle reaches surface of the earth. The velocity of the particle when it reaches the surface of the earth is

$$v = (2gR)^{\frac{1}{2}}$$

Therefore

$$2gR = \frac{g(A^2 - R^2)}{R}$$

$$2R^2 = A^2 - R^2$$

$$A = \sqrt{3}R$$

Let $t=t_1$ and $t=t_2$ be two successive instants when value of x_t changes from +R to –R and therefore $t_2 - t_1$is the time taken to cover the length o the tunnel. But

$$R = \sqrt{3}R\sin\omega t_1$$

$$\sin\omega t_1 = \frac{1}{\sqrt{3}}$$

$$\omega t_1 = \sin^{-1}\frac{1}{\sqrt{3}} = 0.615 \text{ rad}$$

and

$$-R = \sqrt{3}R\sin\omega t_2$$

$$\sin\omega t_2 = -\frac{1}{\sqrt{3}}$$

$$\omega t_2 = -0.615\ rad$$

$$\omega t_1 - \omega t_2 = 0.615 * 2 = 1.23 \text{ rad}$$

$$t_2 - t_1 = \frac{1.23}{\omega} = 1.23\left(\frac{R}{g}\right)^{\frac{1}{2}}$$

c) In this case the particle is thrown up with velocity $(gR)^{\frac{1}{2}}$ it comes back to the surface of the earth with the same speed and then covers the length of the tunnel in the same time as in case (a) above.