Answer on Question #51623, Physics, Mechanics | Kinematics | Dynamics

A boy and his father run together. Mass of the boy is half of his father; kinetic energy of father is half of his son. If the father increases his velocity 1 m/s, then the kinetic energy becomes equal of each. What is their initial velocity?

If they run with their own kinetic energy on a road which is inclined and the road makes an angle of 30 degree with horizontal, how much distance will each of them complete?

Solution:

We start with the given data. Let the father's mass be M, so that the son's mass must be equal to $\frac{1}{2}\mathrm{M}$. We put the father's velocity as V_f and the son's velocity as V_s.

We know that the equation to find kinetic energy, KE, is the following, where m is mass and v is velocity:

$$\mathrm{KE} = \frac{\mathrm{mv}^2}{2}$$

Based on the above information we can apply this to our problem.

$$\frac{\mathrm{m_f V_f^2}}{2} = \frac{1}{2}\left(\frac{\mathrm{m_f V_s^2}}{2}\right)$$

From the noted equation we can express the value of $\mathrm{V_f^2}$. Firstly we simplify the equation.

$$\frac{\mathrm{V_f^2}}{2} = \frac{1}{2}\left(\frac{\mathrm{V_s^2}}{2}\right)$$

We multiply both sides of the equation by 2 and obtained the following result.

$$\mathrm{V_f^2} = \frac{1}{2}\mathrm{V_s^2}$$

Now we have to construct the equation which takes into account the following condition, the father increases his velocity 1 m/s and the kinetic energy becomes equal of each.

$$\frac{1}{2}\left(\frac{\mathrm{m_f(V_f+1)^2}}{2}\right) = \left(\frac{\mathrm{m_f V_f^2}}{2}\right)$$

Simplify the equation by opening the parenthesis.

$$\frac{1}{2}\mathrm{V_f^2} = \frac{1}{4}(\mathrm{V_f^2} + 2\mathrm{V_f} + 1)$$

Now we simplify by opening the parenthesis and combining like terms.

$$\frac{1}{2}\mathrm{V_f^2} - \frac{1}{4}\mathrm{V_f^2} - \frac{1}{2}\mathrm{V_f} - \frac{1}{4} = 0$$

Then we need to solve the obtained quadratic equation for $\mathrm{V_f}$.

$$\frac{1}{4}V_f^2 - \frac{1}{2}V_f - \frac{1}{4} = 0$$

Multiply all terms by 4.

$$V_f^2 - 2V_f - 1 = 0$$

$$V_{f_{1,2}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We determine the first root.

$$V_{f_1} = \frac{2 + \sqrt{(2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 + \sqrt{8}}{2} = 1 + \sqrt{2} = 2.4142 \text{ m/s}$$

We know that the second root will have the negative sign, thus, we accept the first solution.

$$V_f = 2.4142 \text{ m/s}$$

Now we can calculate the son's velocity.

$$V_s = 2 \cdot (2.4142) = 4.828 \text{ m/s}$$

The next part of the task is to determine the distance if we know their kinetic energy on a road which is inclined and the road makes an angle of 30 degree with horizontal.

Firstly we determine the father's kinetic energy.

$$KE = \frac{m_f V_f^2}{2} = \frac{m_f (2.414)^2}{2} = 2.914 m_f$$

The distance to be held father will be equal

$$\text{Distance} = \frac{KE}{m_f g \sin\alpha} = \frac{\frac{m_f V_f^2}{2} \cdot}{m_f g \sin\alpha} = \frac{m_f V_f^2}{2\, m_f g \sin\alpha} = \frac{V_f^2}{2\, g \sin\alpha} = \frac{(2.414)^2}{10} =$$

$$= \frac{5.827}{10} = 0.58 \text{ m}$$

Similarly we determine the distance for the son.

$$\text{Distance} = \frac{V_s^2}{2\, g \sin\alpha} = \frac{(4.828)^2}{10} = \frac{23.310}{10} = 2.3 \text{ m}$$