

Answer on Question 51622, Physics, Mechanics | Kinematics | Dynamics

Question:

The sum of two unit vector is also a unit vector. Then magnitude of their difference is

- 1) 0
- 2) $\sqrt{2}$
- 3) $\sqrt{3}$
- 4) $\sqrt{7}$

Solution:

Let \mathbf{a} and \mathbf{b} be a unit vectors, therefore their magnitudes will be $\|\mathbf{a}\| = \|\mathbf{b}\| = 1$. Because $\mathbf{a} + \mathbf{b}$ has unit length we get:

$$\begin{aligned}1 &= \|\mathbf{a} + \mathbf{b}\|^2, \\1 &= \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\langle \mathbf{a}, \mathbf{b} \rangle, \\1 &= 2 + 2\langle \mathbf{a}, \mathbf{b} \rangle, \\ \langle \mathbf{a}, \mathbf{b} \rangle &= -\frac{1}{2}.\end{aligned}$$

Here, $\langle \mathbf{a}, \mathbf{b} \rangle$ is the dot product or a scalar product of two vectors \mathbf{a} and \mathbf{b} , and we need it in order to obtain the magnitude of their difference:

$$\begin{aligned}\|\mathbf{a} - \mathbf{b}\|^2 &= \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\langle \mathbf{a}, \mathbf{b} \rangle, \\ \|\mathbf{a} - \mathbf{b}\|^2 &= 2 - 2 \cdot \left(-\frac{1}{2}\right), \\ \|\mathbf{a} - \mathbf{b}\|^2 &= 3, \\ \|\mathbf{a} - \mathbf{b}\| &= \sqrt{3}.\end{aligned}$$

Therefore, we get that the magnitude of their difference is $\sqrt{3}$.

Answer:

- 3) $\sqrt{3}$

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