Answer on Question #51562, Physics, Solid State Physics

The Debye temperature for silver is 225 K. Calculate the highest possible frequency for lattice vibrations in silver and its molar heat capacity at 10 K and 500 K.

Solution:

The maximum frequency of vibration of the atoms of solid bodies is given by Eq.(1)

$$f_D = \frac{k_B T_D}{h} = \frac{1.38 \cdot 10^{-23} J/K \cdot 225K}{6.62 \cdot 10^{-34} J \cdot s} = 4.7 \cdot 10^{12} Hz \quad (1)$$

where $k_B = 1.38 \cdot 10^{-23} J/K$ is the Boltzmann constant; $h = 6.62 \cdot 10^{-34} J \cdot s$ is the Planck constant; T_D is the Debye temperature.

The molar heat capacity is given by Eq.(2)

$$C_V(T) = 9Nk_B \left(\frac{T}{T_D}\right)^3 \int_0^{T_D/T} \frac{\xi^4 e^\xi}{\left(e^\xi - 1\right)^2} d\xi \quad (2)$$

where N is the number of atoms in a solid body.

Than

$$C_V(T=10) = 9\left(\frac{10}{225}\right)^3 6.02 \cdot 10^{23} \cdot 1.38 \cdot 10^{-23} J/K \left(\int_0^{225/10} \frac{\xi^4 e^\xi}{\left(e^\xi - 1\right)^2} d\xi\right) = 0.17 J/K$$

$$C_V(T=500) = 9\left(\frac{500}{225}\right)^3 6.02 \cdot 10^{23} \cdot 1.38 \cdot 10^{-23} J/K \left(\int_0^{225/500} \frac{\xi^4 e^\xi}{\left(e^\xi - 1\right)^2} d\xi\right) = 26.67 J/K$$

Answer: $f_D = \frac{k_B T_D}{h} = 4.7 \cdot 10^{12} Hz$; $C_V(T=10) = 0.17 J/K$; $C_V(T=500) = 26.67 J/K$