

### Answer on Question #51559, Physics, Solid State Physics

Show that for a square lattice in two dimensions the kinetic energy of a free electron at a corner of the first Brillouin zone is higher than that of an electron at the midpoint of a side face of the zone by a factor of 2.

**Answer:**

The first Brillouin zone is therefore the square  $\left[-\frac{\pi}{a}, \frac{\pi}{a}\right] \times \left[-\frac{\pi}{a}, \frac{\pi}{a}\right]$ .

A corner of the zone is  $(k_x, k_y) = \left(\pm \frac{\pi}{a}, \pm \frac{\pi}{a}\right)$ .

A free electron with one of these four wavevectors has kinetic energy

$$E_{\text{CORNER}} = \frac{\hbar^2}{2m} \vec{k}^2 = \frac{\hbar^2 \pi^2}{ma^2}.$$

The faces of the wave zone have one of  $k_x, k_y$  with magnitude  $\pi/a$  and the other component zero, giving  $E_{\text{FACE}} = \frac{\hbar^2}{2m} (\pi/a)^2 = E_{\text{CORNER}} / 2$ .