Answer on Question #51544, Physics, Other

6 Given three vectors

$\vec{a} = -\vec{i} - 4\vec{j} + 2\vec{k}$, $\vec{b} = 3\vec{i} + 2\vec{j} - 2\vec{k}$, $\vec{c} = 2\vec{i} - 3\vec{j} + \vec{k}$, calculate $\vec{a} \cdot (\vec{b} \times \vec{c})$

A −6

B 6

C 9

D −9

Solution:

We can define the cross product $(\vec{b} \times \vec{c})$ by the determinant of the matrix:

$$\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & b & c \\ d & e & f \end{pmatrix}$$

We can compute this determinant as

$$\begin{pmatrix} b & c \\ e & f \end{pmatrix}\mathbf{i} - \begin{pmatrix} a & c \\ d & f \end{pmatrix}\mathbf{j} + \begin{pmatrix} a & b \\ d & e \end{pmatrix}\mathbf{k}$$

$$= (bf - ce)\vec{i} + (cd - af)\vec{j} + (ae - bd)\vec{k}$$

In our case

$$\left|\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & -2 \\ 2 & -3 & 1 \end{pmatrix}\right| = \big(2*1 - (-2)*(-3)\big)\vec{i} + (-2*2 - 3*1)\vec{j} + (3*(-3) - 2*2)\vec{k} =$$

$$= -4\vec{i} - 7\vec{j} - 13\vec{k}$$

Dot Product:

$$a \cdot b = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$$

In our case:

$$a \cdot (b \times c) = (-1) \cdot (-4) + (-4) \cdot (-7) + 2 \cdot (-13) = 4 + 28 - 26 = 6$$

Answer: B. 6