Answer on Question #51535-Physics-Mechanics-Kinematics-Dynamics

A ring of radius r is to be mounted on a wheel of radius R. The coefficient of linear expansion of the material of the ring is α. Young's modulus is Y, area of cross section is A and mass is m. Initially ring and wheel are at same temperature. (r<R)

a) The temperature through which the ring should be heated so that it can be mounted on the wheel

b) Suppose the wheel with mounted ring starts rotating with angular velocity ω. The value of ω for which tension in ring becomes zero

Solution

a) The circumference of a thin ring can be expressed as

$$c_0 = 2\pi r_0$$

where c_0 is initial circumference, r_0 is initial radius.

The change in circumference due to temperature change can be expressed as

$$\Delta c = c_1 - c_0 = 2\pi r_0 \, \Delta T \alpha$$

where Δc is change in circumference, c_1 is final circumference, ΔT is temperature change, α is linear expansion coefficient.

The final circumference can be expressed as

$$c_1 = 2\pi r_1$$

where r_1 is final radius.

So,

$$\Delta c = 2\pi r_0 \, \Delta T \alpha = 2\pi r_1 - 2\pi r_0.$$

Thus

$$r_1 = r_0(1 + \alpha \Delta T) \; or \; \Delta T = \frac{r_1 - r_0}{\alpha r_0}.$$

In our case $r_1 = r, r_2 = R$:

$$\Delta T = \frac{R - r}{\alpha r}.$$

b) Change in length of the ring is

$$\Delta c = 2\pi R - 2\pi r.$$

Longitudinal strain is

$$\frac{2\pi R - 2\pi r}{2\pi r} = \frac{R - r}{r}.$$

$$Y = \frac{\frac{F}{A}}{\frac{R - r}{r}} \to F = \frac{YA(R - r)}{r}.$$

Tension in ring becomes zero if

$$F = F_{rotational} = m\omega^2 R.$$

Thus,

$$\frac{YA(R-r)}{r} = m\omega^2 R.$$

$$\omega = \sqrt{\frac{YA(R-r)}{mrR}}.$$