Answer on Question#51456 - Physics - Mechanics - Kinematics - Dynamics

1. A particle moves on a circle in accordance to the equation $s = t^4 - 8t$, where s is the displacement in feet measured along the circular path and t is in seconds. 2 seconds after starting from rest the total acceleration of the particle is $48\sqrt{2}\frac{\mathrm{ft}}{\mathrm{sec}^2}$. Compute the radius of the circle.

2. As you drive down the road at $v_0 = 17\frac{\mathrm{m}}{\mathrm{s}}$, you press at the gas pedal and speed up with the uniform acceleration of $a = 21.12\frac{\mathrm{m}}{\mathrm{s}^2}$ for $t = .65\mathrm{s}$. If the tires on your car have a radius of $R = 33\mathrm{cm}$ what is their angular displacement during the period of acceleration.

Solution:

1. The speed of the particle at time t is
$$v(t) = \frac{ds}{dt} = 4t^3 - 8$$
The tangent acceleration is at time t is
$$a_\tau(t) = \frac{d^2s}{st^2} = \frac{dv}{dt} = 12t^2$$
The centripetal acceleration at time t is
$$a_r(t) = \frac{v^2(t)}{r} = \frac{(4t^3 - 8)^2}{r}$$
Since centripetal and tangent accelerations are perpendicular to each other, the total acceleration is given by
$$a(t) = \sqrt{a_\tau^2(t) + a_r^2(t)} = \sqrt{144t^4 + \frac{(4t^3 - 8)^4}{r^2}}$$
It's given that $a(2) = 48\sqrt{2}\frac{\mathrm{ft}}{\mathrm{s}^2}$, so
$$\sqrt{144 \cdot 2^4 + \frac{(4 \cdot 2^3 - 8)^4}{r^2}} = 48\sqrt{2}$$
$$r = 12\mathrm{ft}$$
2. The distance traveled during the period of acceleration is
$$l = v_0 \cdot t + \frac{a \cdot t^2}{2}$$
To find the angular displacement of tires in radians, we should divide this distance by the radius of tires
$$\Delta\varphi = \frac{l}{R} = \frac{v_0 \cdot t}{R} + \frac{a \cdot t^2}{2R} = \frac{17\frac{\mathrm{m}}{\mathrm{s}} \cdot 0.65\mathrm{s}}{0.33\mathrm{m}} + \frac{21.12\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 0.4225\mathrm{s}^2}{0.66\mathrm{m}} = 47$$

Answer:

1. $r = 12\mathrm{ft}$
2. $\Delta\varphi = \frac{v_0 \cdot t}{R} + \frac{a \cdot t^2}{2R} = 47$