Answer on Question #51446, Physics, Solid State Physics

Write down the wave function ψ_{211} for the hydrogen atom. Calculate the probability current density for this state.

Solution:

The wave function ψ_{211} 1 for the hydrogen atom is given by Eq.(1)

$$\psi_{211} = \frac{1}{2\sqrt{6a_0^3}} \cdot \frac{r}{a_0} \exp\left[-\frac{r}{2a_0}\right] \sqrt{\frac{3}{8\pi}} \sin\theta e^{i\varphi} \qquad (1)$$

The probability current density for this state is given by Eq.(2).

$$\vec{\mathrm{j}} = \frac{\mathrm{i}e\hbar}{2\mathrm{m}_0}\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right) \qquad (2)$$

So,

$$\mathrm{j}_\mathrm{r} = \frac{\mathrm{i}e\hbar}{2\mathrm{m}_0}\left(\psi\frac{\partial\psi^*}{\partial \mathrm{r}} - \psi^*\frac{\partial\psi}{\partial \mathrm{r}}\right) = 0$$

$$\mathrm{j}_\theta = \frac{\mathrm{i}e\hbar}{2\mathrm{m}_0}\frac{1}{\mathrm{r}}\left(\psi\frac{\partial\psi^*}{\partial\theta} - \psi^*\frac{\partial\psi}{\partial\theta}\right) = 0$$

$$\mathrm{j}_\varphi = \frac{\mathrm{i}e\hbar}{2\mathrm{m}_0}\frac{1}{\mathrm{r}\sin\theta}\left(\psi\frac{\partial\psi^*}{\partial\varphi} - \psi^*\frac{\partial\psi}{\partial\varphi}\right) =$$

$$\frac{\mathrm{i}e\hbar}{2\mathrm{m}_0}\frac{1}{\mathrm{r}\sin\theta}\frac{1}{2\sqrt{6\mathrm{a}_0^3}} \cdot \frac{\mathrm{r}}{\mathrm{a}_0}\exp\left[-\frac{\mathrm{r}}{2\mathrm{a}_0}\right]\sqrt{\frac{3}{8\pi}}\sin\theta\left(-\mathrm{i}e^{\mathrm{i}\varphi}e^{-\mathrm{i}\varphi} - \mathrm{i}e^{-\mathrm{i}\varphi}e^{\mathrm{i}\varphi}\right) =$$

$$= \frac{e\hbar}{2\mathrm{a}_0\mathrm{m}_0\sqrt{6\mathrm{a}_0^3}} \cdot \exp\left[-\frac{\mathrm{r}}{2\mathrm{a}_0}\right]\sqrt{\frac{3}{8\pi}}$$

Answer: $\dfrac{e\hbar}{2\mathrm{a}_0\mathrm{m}_0\sqrt{6\mathrm{a}_0^3}} \cdot \exp\left[-\dfrac{\mathrm{r}}{2\mathrm{a}_0}\right]\sqrt{\dfrac{3}{8\pi}}$