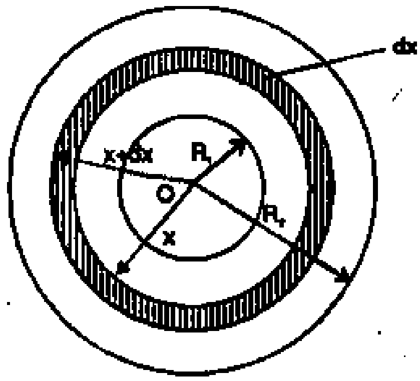
Answer on Question #51386-Physics-Mechanics-Kinematics-Dynamics

The moment of inertia of a annular disc of mass M, the inner diameter R_1 and outer diameter is R_2 about a perpendicular axis through its center is

Solution



An annular disc is just a circular disc from which a smaller, concentric disc has been removed, leaving a hole in its place as shown in the figure.

Let O be the center of annular disc. Then, clearly, face area of the disc is $\pi(R_2^2 - R_1^2)$ and, therefore,

mass per unit area of the disk $= \frac{M}{\pi(R_2^2 - R_1^2)}$.

Now, consider a coaxial ring of radius x and width dx shown in the figure. We clearly have,

face area of the ring $= 2\pi x dx$ and, therefore, it mass $= \frac{M}{\pi(R_2^2 - R_1^2)} 2\pi x dx = \frac{M2x}{(R_2^2 - R_1^2)} dx$.

Hence, the moment of inertia of this ring about an axis through O and perpendicular to its plane is

$$\frac{M2x}{(R_2^2 - R_1^2)} dx \cdot x^2 = \frac{M2x^3}{(R_2^2 - R_1^2)} dx.$$

The moment of inertia of the whole annular disk about an axis through O and perpendicular to its plane is thus easily obtained by integrating the above expression for the moment of inertia of this ring, between the limits $x = R_1$ and $x = R_2$, so that, we have

$$I = \int_{R_1}^{R_2} \frac{M2x^3}{(R_2^2 - R_1^2)} dx = \frac{2M}{(R_2^2 - R_1^2)} \int_{R_1}^{R_2} x^3 dx = \frac{2M}{(R_2^2 - R_1^2)} \left[\frac{x^4}{4}\right]_{R_1}^{R_2} = \frac{2M}{(R_2^2 - R_1^2)} \left[\frac{(R_2^4 - R_1^4)}{4}\right]$$
$$= \frac{M(R_2^2 + R_1^2)}{2}.$$

Answer: $\frac{M(R_2^2 + R_1^2)}{2}$.