Answer on Question #51385-Physics-Mechanics-Kinematics-Dynamics

A certain sprinter has a top speed of 10.6 m/s. If the sprinter starts from rest and accelerates at a constant rate, he is able to reach his top speed in a distance of 10.1 m. He is then able to maintain his top speed for the remainder of a 100 m race.

(a) What is his time for the 100 m race?

(b) In order to improve his time, the sprinter tries to decrease the distance required for him to reach his top speed. What must this distance be if he is to achieve a time of 10.3 s for the race?

Solution

(a) An average velocity of sprinter on acceleration part is

$$\bar{v} = \frac{1}{2}\left(v_{initial} + v_{final}\right) = \frac{1}{2}\left(0\frac{m}{s} + 10.6\frac{\mathrm{m}}{\mathrm{s}}\right) = 5.3\frac{m}{s}.$$

Therefore it took a time of

$$t_1 = \frac{s_1}{\bar{v}} = \frac{10.1\ \mathrm{m}}{5.3\frac{m}{s}} = 1.9\ s$$

to complete this distance .

He ran the remaining $100\ m - 10.1\ m = 89.9\ m$ at a speed of $10.6\frac{\mathrm{m}}{\mathrm{s}}$, or in a time of

$$t_2 = \frac{s_2}{v} = \frac{89.9\ m}{10.6\frac{\mathrm{m}}{\mathrm{s}}} = 8.5\ s.$$

His total time for the race is

$$t = t_1 + t_2 = 1.9\ s + 8.5\ s = 10.4\ s$$

(b) Let's call t_1 the time spent in the acceleration phase, then $(10.3 - t_1)$ is the time spent in the remainder of the race.

Since his maximal speed is still $10.6\frac{\mathrm{m}}{\mathrm{s}}$, his average speed in the acceleration part will still be $5.3\frac{m}{s}$; in the time t_1, he will cover a distance of $5.3t_1$ meters; in the rest of the race he will cover a distance of $10.6(10.3 - t_1)$ meters. The sum of these distances is $100m$, so we have

$$5.3t_1 + 10.6(10.3 - t_1) = 100.$$

We can solve this easily for t_1:

$$t_1 = 1.73s$$

This means he must complete the acceleration phase in $1.73s$; running at an average velocity of $5.3\frac{m}{s}$ for $1.73s$ means he covers a distance of

$$5.3\frac{m}{s} \cdot 1.73s = 9.17\ m.$$

before reaching his constant speed of $10.6\frac{\mathrm{m}}{\mathrm{s}}$.

Answer: (a) $\mathbf{10.4\ s}$; (b) $\mathbf{9.17\ m}$.