A lead ball is dropped in a lake from a diving board 7.32 m above the water. It hits the water with a certain velocity and then sinks to the bottom with the same constant velocity. It reaches the bottom 4.88 s after it is dropped. (a) How deep is the lake?

Solution.

We right equation of motion of a lead ball:

$$m\,d^2/dt^2\,x=-mg, h_0>x>h_w \quad ,$$

$$m\,d^2/dt^2\,x=0, h_w>x>0,$$

where h_0 is the initial height of a ball, and h_w is the water level above the bottom. Solving these equations we have

$$x=-g\,t^2/2+v_0\,t+h_{0,}\,h_0>x>h_w,$$

$$x=v_1\,t+h_w, h_w>x>0,$$

where $v_0=0\,m/s$ and v_1 can be find from the energy conservation law

$$mv_1^2/2=mg(h_0-h_w)$$

$$v_1=\sqrt{2\,g(h_0-h_w)}$$

Thus

$$x=-g\,t^2/2+h_{0,}\,h_0>x>h_w,$$

$$x=\sqrt{2\,g(h_0-h_w)}\,t+h_w, h_w>x>0,$$

Total time when ball falls

$$t_{tot}=t_1+t_2$$

$$h_0-h_w=g\,t_1^2/2$$

$$h_w=\sqrt{2\,g(h_0-h_w)}\,t_2 \quad ,$$

$$\sqrt{2(h_0-h_w)/g}=t_1$$

$$h_w/\sqrt{2\,g(h_0-h_w)}=t_2 \quad ,$$

$$\sqrt{2(h_0-h_w)/g}+h_w/\sqrt{2\,g(h_0-h_w)}=t_{tot}$$

$$h_w=\sqrt{2\,g(h_0-h_w)}\left(t_{tot}-\sqrt{2(h_0-h_w)/g}\right)$$

$$h_w=\sqrt{2*9.8\,m/s^2(7.32\,m)}\left(4.88\,s-\sqrt{2(7.32\,m)/9.8\,m/s^2}\right)$$

Answer.

$$h=43.8\,m$$

(b) What is the magnitude of the average velocity of the ball for the entire fall?

Solution.

We calculate two time periods:

$$\sqrt{2(h_0-h_w)/g}=t_1=1.22\,s$$

$$t_2=4.88\,s-1.22\,s=3.66\,s$$

For the first period average velocity is a half of maximum velocity:

$$\langle v_1\rangle=v_1/2=\sqrt{2\,g(h_0-h_w)}/2=11.98/2\,m/s=5.99\,m/s$$

$$\langle v_2\rangle=11.98\,m/s$$

$$\langle v\rangle=\langle v_1\rangle t_1/t_{tot}+\langle v_2\rangle t_2/t_{tot}=5.99\,m/s*1.22\,s/4.88\,s+11.98\,m/s*3.66\,s/4.88\,s=10.48\,m/s$$

Answer.

$$\langle v\rangle=10.48\,m/s$$

(c) Suppose the water is drained from the lake. The ball is now thrown from the diving board so that it again reaches the bottom in 4.88 s. What is the magnitude of the initial velocity of the ball?

Solution.

We right equation of motion of a lead ball:

$$m\,d^2/dt^2\,x=-mg, h_0>x>0 \quad ,$$

at the endpoint of the trajectory we have

$0=-gt^2/2+v_0t+h_0$

$v_0=(9.8\,m/s^2(4.88\,s)^2/2-43.8\,m-7.32\,m)/4.88\,s$

Answer.

$v_0=13.44\,m/s$