To stop a car, first you require a certain reaction time to begin braking; then the car slows under the constant braking deceleration. Suppose that the total distance moved by your car during these two phases is 51.9 m when its initial speed is 78.3 km/h, and 23.6 m when its initial speed is 49.1 km/h. What is (a) your reaction time?

Solution.

For both cases (initial velocities v_1 and v_2) we right down Newton's equations:

$d^2/t^2\, x=0, 0<t<t_{0,}$

$d^2/t^2\, x=-a_s, t_0<t,$

where t_0 is the reaction time and a_s is the deceleration value. We solve these equations for different velocity values:

$x=v_1 t, 0<t<t_{0,}$

$x=-at^2/2+v_1 t, t_0<t$

for $v=v_2$

$x=v_2 t, 0<t<t_{0,}$

$x=-at^2/2+v_2 t, t_0<t$.

Since reaction time t_0 in both cases is the same, we introduce the length of the path before driver's reaction

$l_{1,2}=v_{1,2}t_0$.

Deceleration continues until car stops. We find duration of deceleration requiring

$dx/dt=v\left(t_{d\,1,2}\right)=0$

$v\left(t_{d\,1,2}\right)=-a\left(t_{d\,1,2}\right)+v_{1,2}=0$

$t_{d\,1,2}=v_{1,2}/a$

Putting it into the main equation, we obtain

$$d_{1,2}=v_{1,2}t_0+v\frac{v_{1,2}}{a}-\frac{a}{2}\left(\frac{v_{1,2}}{a}\right)^2$$

$$d_{1,2}=v_{1,2}t_0+\frac{v_{1,2}^2}{2a}$$

$$\frac{1}{2}\frac{v_{1,2}^2}{d_{1,2}-v_{1,2}t_0}=a$$

$$\frac{v_1^2}{d_1-v_1 t_0}=\frac{v_2^2}{d_2-v_2 t_0}$$

$$\frac{d_1-v_1 t_0}{v_1^2}=\frac{d_2-v_2 t_0}{v_2^2}$$

$$t_0=\frac{v_1^2 d_2-v_2^2 d_1}{v_1 v_2^2-v_2 v_1^2}=\frac{(78.3\,km/h)^2 23.6\,m-(49.1\,km/h)^2 59.9\,m}{(78.3\,km/h)(49.1\,km/h)^2-(49.1\,km/h)(78.3\,km/h)^2}$$

$$t_0=\frac{(21.75\,m/s)^2 23.6\,m-(13.64\,m/s)^2 59.9\,m}{(21.75\,m/s)(13.64\,m/s)^2-(13.64\,m/s)(21.75\,m/s)^2}=-0.008\,s$$

Answer.

$t_0=-0.008\,s$

What is (b) the magnitude of the deceleration?

Solution.

$$a=\frac{1}{2}\frac{v_{1,2}^2}{d_{1,2}-v_{1,2}t_0}=0.5\frac{(21.75\,m/s)^2}{23.6\,m-21.75\,m/s*0.008\,s}=10.1\,m/s^2$$

Answer.

$a=10.1\,m/s^2$