Answer on Question #51375, Physics, Mechanics Kinematics Dynamics

In an arcade video game, a spot is programmed to move across the screen according to $x = 8.79\text{t} - 0.658\text{t}^3$, where x is the distance in centimeters measured from the left edge of the screen and t is time in seconds. When the spot reaches a screen edge, either at $x = 0$ or $x = 15.0cm$, t is reset to 0 and the spot starts moving again according to x(t).

(a) At what time after starting is the spot instantaneously at rest?

(b) At what value of x does this occur?

(c) What is the spot's acceleration when this occurs?

(d) At what time t > 0 does the spot first reach an edge of the screen?

Solution:

(A) The spot is instantaneously at rest if $x = 0$ or $x = 15.0cm$. Than if $x = 0$
$8.79\text{t} - 0.658\text{t}^3 = 0 \Rightarrow t\left(8.79 - 0.658\text{t}^2\right) = 0$

$$t_1 = 0s$$

$$t_{2,3} = \pm\sqrt{\frac{8.79}{0.658}} = \pm 3.65s$$

We consider only physically correct solutions ($t > 0$).

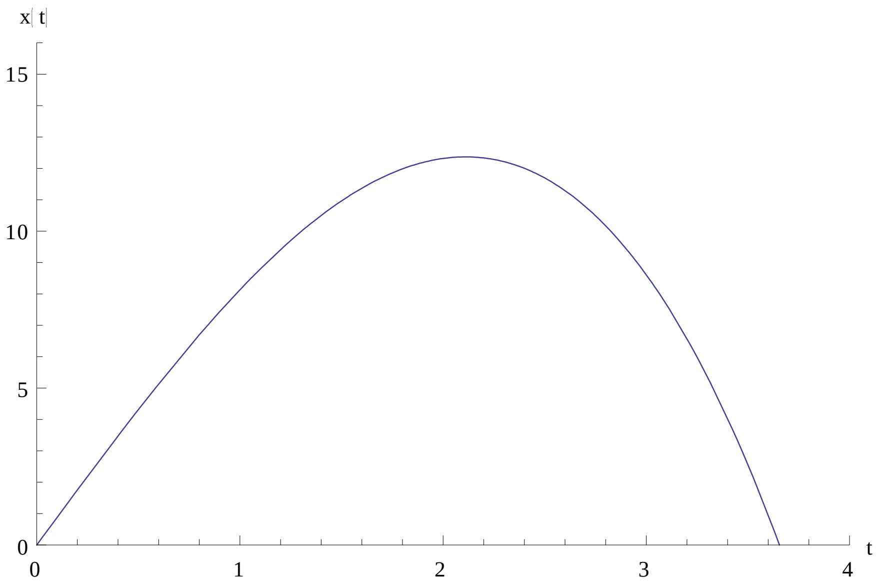

Fig.1

If $x = 15.0cm$ than $8.79\text{t} - 0.658\text{t}^3 = 15$

We built the dependence of $x(t)$ using mathematical software (see Fig.1). From Fig.1 it is clear that x never get 15cm.

(B) From Fig. 1 it clear that $x \in [0, x_{\max}]$. So $\frac{dx}{dt} = 8.79 - 3 \cdot 0.658 t^2 = 0 \Rightarrow t = 2.11$, than

$$\frac{d^2x}{dt^2} = -6 \cdot 0.658 t \Rightarrow \frac{d^2x}{dt^2}(2.11) = -6 \cdot 0.658 \cdot 2.11 = -8.33 < 0 \Rightarrow t_{\max} = 2.11.$$

$$x_{\max}(2.11) = 8.79 \cdot 2.11 - 0.658 \cdot 2.11^3 = 12.37 cm$$

$$x \in [0, 12.37]$$

(C) The spot's acceleration is $a(t) = \frac{d^2x}{dt^2} = -6 \cdot 0.658 t$

$$a(0) = 0$$

$$a(3.65) = -6 \cdot 0.658 \cdot 3.65 = -14.41 m/s$$

(D) The spot is never reach an edge of the screen (see Fig.1)