Answer on Question #51372 – Physics, Mechanics – Kinematics – Dynamics:

An object falls a distance h from rest. If it travels $0.55h$ in the last 1.00 s, find (a) the time and (b) the height of its fall.

Solution.

Let t be the time of fall. We have that free fall is a uniformly accelerated motion with an acceleration of gravity. So:

$$h = \frac{gt^2}{2};$$

An object travels $0.55h$ in the last 1.00 s, so it travels $0.45h$ in the first $t - 1$ s. Hence:

$$0.45h = \frac{g(t-1)^2}{2} \Longrightarrow h = \frac{g(t-1)^2}{0.9};$$

So we have an equation with respect to t:

$$\frac{gt^2}{2} = \frac{g(t-1)^2}{0.9} \Longrightarrow \frac{t^2}{2} = \frac{(t-1)^2}{0.9} \Longrightarrow \frac{9t^2}{20} = (t-1)^2 \Longrightarrow \frac{3t}{\sqrt{20}} = t - 1 \Longrightarrow$$

$$\Longrightarrow \frac{3t}{2\sqrt{5}} = t - 1 \Longrightarrow \left(1 - \frac{3}{2\sqrt{5}}\right)t = 1 \Longrightarrow t = \frac{1}{1 - \frac{3}{2\sqrt{5}}} \Longrightarrow t = \frac{2\sqrt{5}}{2\sqrt{5} - 3} \Longrightarrow$$

$$\Longrightarrow t = \frac{2\sqrt{5}\left(2\sqrt{5} + 3\right)}{11} \Longrightarrow t = \frac{20 + 6\sqrt{5}}{11} \approx 3.04 \text{ s};$$

Now find h. Assume that $g = 9.8\ m/s^2$:

$$h = \frac{gt^2}{2} = \frac{9.8 \cdot \left(20 + 6\sqrt{5}\right)^2}{2 \cdot 121} \approx 45.22 \text{ m}.$$

Answer.

$t = 3.04$ s, $h = 45.22$ m.