Answer on Question #51296-Physics-Electromagnetism

Obtain a relation between the refractive index and polarizability of a dielectric material.

Solution

Consider a local field $\vec{F}$ inside the dielectric and its relation to an applied field $\vec{E}$. The Lorentz local field considers a spherical region inside a dielectric that is large compared to the size of a molecule. The field inside this uniformly polarized sphere behaves as if it were due to a dipole given by:

$$\vec{\mu} = \frac{4\pi a^3}{3}\vec{P}$$

Since $\vec{P}$ is the polarization per unit volume and $\frac{4\pi a^3}{3}$ is the volume of the sphere we see that μ is the induced dipole moment or polarization (these are equivalent). The local field is the macroscopic field E minus the contribution of the due to the matter in the sphere:

$$\vec{F} = \vec{E} - \overrightarrow{E_{int}} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}.$$

Since $\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$ the Lorentz local field is

$$\vec{F} = \frac{(\varepsilon_r + 2)}{3}\vec{E}.$$

Since $\varepsilon_r = 1$ for vacuum and $\varepsilon_r > 1$ for all dielectric media it is apparent that the local field is always larger than the applied field. This simple consequence of the theory of dielectric polarization causes confusion. We usually think of the dielectric constant as providing a screening of the applied field. Therefore, we might be inclined to think of a local field as smaller than the applied field. However, this naïve view ignores the role of the polarization of the dielectric itself. Inside the sphere we have carved out of the dielectric we observe the macroscopic (applied) field plus the field due to the polarization of the medium. The sum of these two contributions leads to a field that is always larger than the applied electric field.

The polarization is the number density times the polarizability times the local field.

$$\vec{P} = \frac{N\alpha}{V}\vec{F} = \frac{N\alpha}{3V}(\varepsilon_r + 2)\vec{E} = \varepsilon_0(\varepsilon_r - 1)\vec{E}.$$

We eliminate $\vec{E}$ to obtain the Clausius-Mossotti equation.

$$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha}{3V\varepsilon_0}$$

This equation connects the macroscopic dielectric constant ε_r to the microscopic polarizability. Since $\varepsilon_r = n^2$ we can replace these to obtain the Lorentz-Lorentz equation:

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha}{3V\varepsilon_0}.$$