Answer on Question #51292, Physics, Mechanics | Kinematics | Dynamics

Consider two different radio station which broadcast signals whose angular frequency differs by f hertz. They are received by a receiver at origin. Determine the time for which receiver remains idle in each cycle if it can detect amplitude more that 1.5 A only.

Solution:

Consider the superposition of two signals of different frequencies

$$A\,(t) = A_0\,cos(\omega_1 t) + A_0\,cos(\omega_2 t)$$

The addition of the two cosines now gives

$$A(t) = 2A_0 cos(\frac{\omega_1 t - \omega_2 t}{2}) cos(\frac{\omega_1 t + \omega_2 t}{2})$$

i.e.

$$A(t) = 2A_0 cos\left(\frac{(\omega_1 - \omega_2)t}{2}\right) cos\left(\frac{(\omega_1 + \omega_2)t}{2}\right)$$

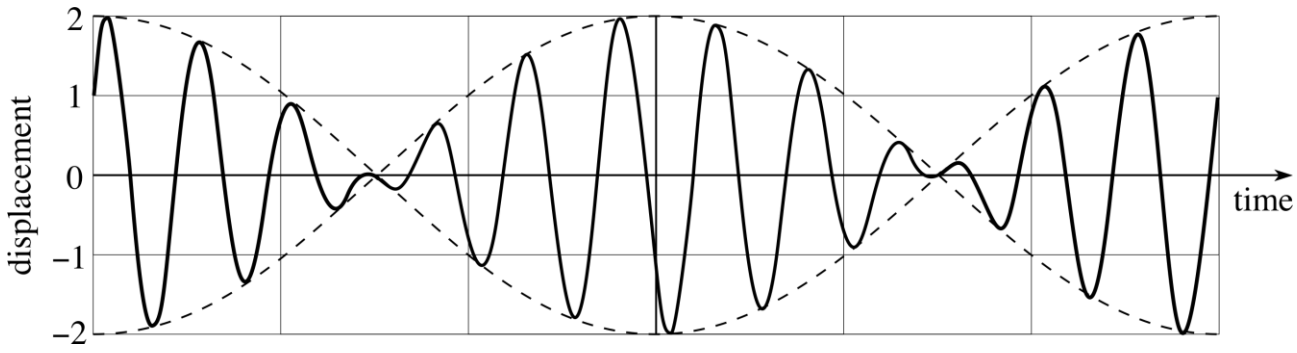
From given

$$\omega_1 - \omega_2 = f$$

Hence

$$A(t) = 2A_0 cos\left(\frac{ft}{2}\right) cos\left(\frac{(\omega_1 + \omega_2)t}{2}\right)$$

In this case $|\,\omega1 - \omega2\,| \ll (\omega1 + \omega2)$ the first cosine function is a slowly varying function of time, as compared with the rapidly varying second cosine function.



The displacement – time graph of this superposition of signals is shown in Figure for the case where the two equal amplitude original oscillations have frequencies in the ratio 13 : 11.

The slow periodic amplitude oscillations are referred to as beats. The beat frequency is equal to $|\,\omega_1 - \omega_2\,|/2\pi$.

The time for which receiver remains idle in each cycle if it can detect amplitude more that $1.5A_0$ only is in period where

$$\left|2A_0 cos\left(\frac{ft}{2}\right)\right| \le 1.5A_0$$

$$\left|cos\left(\frac{ft}{2}\right)\right| \le 0.75$$

$$-0.75 \le cos\left(\frac{ft}{2}\right) \le 0.75$$

From $cos\left(\frac{ft}{2}\right) = -0.75$ we obtain

$$\frac{ft}{2} = \cos^{-1}(-0.75) = 2.419$$

$$t_{-0.75} = 2 * \frac{2.4189}{f} = \frac{4.8378}{f}$$

From $cos\left(\frac{ft}{2}\right) = 0.75$ we obtain

$$t_{0.75} = 2 * \frac{0.7227}{f} = \frac{1.4454}{f}$$

Time is

$$t = t_{-0.75} - t_{0.75} = \frac{1}{f}(4.8378 - 1.4454) = \frac{3.3924}{f}$$

Answer: $t = \frac{3.3924}{f}$