Answer on Question #51240, Physics, Solid State Physics

Show that for a square lattice in two dimensions the kinetic energy of a free electron at a corner of the first Brillouin zone is higher than that of an electron at the midpoint of a side face of the zone by a factor of 2.

Solution:

The term "first zone" must be referring to the first Brillouin zone in k-space. The reciprocal lattice of a square space lattice is also a square lattice. For a real space (i.e., crystal) lattice of side a, the reciprocal lattice points are separated by steps of $2\pi/a$ in the k_X and k_Y directions. The Wigner-Seitz cells in reciprocal space are squares also. These have boundaries that bisect the segments connecting reciprocal lattice points. The first Brillouin zone is therefore the square $\left[-\frac{\pi}{a},\frac{\pi}{a}\right]\times\left[-\frac{\pi}{a},-\frac{\pi}{a}\right]$. A corner of the zone is $(k_X,k_Y)=\left(\pm\frac{\pi}{a},\pm\frac{\pi}{a}\right)$. A free electron with one of these four wavevectors has kinetic energy $E_{CORNER}=\frac{\hbar^2}{2m}\vec{k}^2=\frac{\hbar^2\pi^2}{ma^2}$ The faces of the wave zone have one of k_X, k_Y with magnitude π/a and the other component zero, giving $E_{FACE}=\frac{\hbar^2}{2m}(\pi/a)^2=E_{CORNER}/2$.

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