Answer on Question #51237, Physics, Solid State Physics

The Debye temperature for silver is 225 K. Calculate the highest possible frequency for lattice vibrations in silver and its molar heat capacity at 10 K and 500 K.

Solution:

The maximum frequency of vibration of the atoms of solid bodies

$$\nu_{\max} = \frac{k_B T_D}{h} = \frac{1.38 \cdot 10^{-23} J/K \cdot 225K}{6.62 \cdot 10^{-34} J \cdot s} = 4.7 \cdot 10^{12} Hz \quad (1)$$

where $k_B = 1.38 \cdot 10^{-23} J/K$ is the Boltzmann constant; $h = 6.62 \cdot 10^{-34} J \cdot s$ is the Planck constant; T_D is the Debye temperature.

The molar heat capacity is given by Eq.(2)

$$C_V(T) = 3Nk_B \left(\frac{3}{x_D^3} \int_0^{X_D} \frac{x^4 e^x}{\left(e^x - 1\right)^2} dx \right) \quad (2)$$

where $3N$ is number of normal modes; $x_D = T_D / T$.

Than

$$C_V(10) = 3Nk_B \left(\frac{3}{(225/10)^3} \int_0^{225/10} \frac{x^4 e^x}{\left(e^x - 1\right)^2} dx \right) = 0.0205 Nk_B$$

$$C_V(500) = 3Nk_B \left(\frac{3}{(225/500)^3} \int_0^{225/500} \frac{x^4 e^x}{\left(e^x - 1\right)^2} dx \right) = 2.9698 Nk_B$$

Answer: $\nu_{\max} = \frac{k_B T_D}{h} = 4.7 \cdot 10^{12} Hz$; $C_V(10) = 0.0205 Nk_B$; $C_V(500) = 2.9698 Nk_B$