Answer on Question #51226, Physics, Other

Write down the wave function ψ_{211} for the hydrogen atom. Calculate the probability current density for this state.

Solution:

The wave function ψ_{211} for the hydrogen atom is

$$\psi_{211} = -\frac{1}{8a^2\sqrt{\pi a}} r e^{-\frac{r}{2a}} \sin\theta\, e^{i\phi}$$

The probability current is

$$\mathbf{J} = \frac{i\hbar}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi)$$

The gradient in spherical coordinates is

$$\nabla = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial}{\partial\theta} + \frac{1}{r\sin\theta}\hat{\phi}\frac{\partial}{\partial\phi}$$

so we can calculate $\nabla\psi_{211}$

$$\nabla\psi_{211} = \frac{-1}{8a^2\sqrt{\pi a}} r e^{-\frac{r}{2a}} e^{i\phi}\left[\left(1 - \frac{r}{2a}\right)\sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\theta} + i\hat{\phi}\right]$$

We can now calculate **J**. The contributions along the $\hat{\mathbf{r}}$ and $\hat{\theta}$ directions cancel out, so we are left with

$$\mathbf{J} = \frac{i\hbar}{2m}\left[-\frac{1}{8a^2\sqrt{\pi a}} r e^{-\frac{r}{2a}} \sin\theta\, e^{i\phi} \frac{i}{8a^2\sqrt{\pi a}} e^{-\frac{r}{2a}} e^{-i\phi} - \frac{1}{8a^2\sqrt{\pi a}} r e^{-\frac{r}{2a}} \sin\theta\, e^{-i\phi} \frac{i}{8a^2\sqrt{\pi a}} e^{-\frac{r}{2a}} e^{i\phi}\right]\hat{\phi} =$$

$$= \frac{\hbar}{64\pi m a^5} r e^{-\frac{r}{2a}} \sin\theta\,\hat{\phi}$$

Answer: $\mathbf{J} = \frac{\hbar}{64\pi m a^5} r e^{-\frac{r}{2a}} \sin\theta\,\hat{\phi}$