Answer on Question #51220, Physics, Other

An electron which has a kinetic energy 1.0 MeV collides with a stationary positron. (A positron has a mass equal to an electron but the opposite charge). In the collision both particles annihilate each other releasing two photons of equal energy which travel at an angle of to the electron's direction of motion. Calculate the energy, momentum and angle of emission θ for each photon.

Solution:

The incident electron, with rest mass m = 0.511 MeV/c^2, has momentum p along the positive x-axis and kinetic energy K.

A particle with rest mass m moving with speed v has kinetic energy K given by

$$K = (\gamma - 1)mc^2$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{1 + \left(\frac{p}{mc}\right)^2}$$

We obtain

$$p = \frac{\sqrt{K(K + 2mc^2)}}{c} = \frac{\sqrt{1 * (1 + 2 * 0.511)}}{3 * 10^8} = 1.422 \text{ MeV/c}$$

The total energy E of the electron and the stationary positron before the collision is

$$E = K + 2mc^2 = 1.0 + 2 * 0.511 = 2.022 \text{ MeV}$$

The two photons emerge from the collision each with energy

$$E_\gamma = \frac{E}{2} = 1.011 \text{ MeV}$$

as given by conservation of energy, and, using that the energy E and momentum p of a particle with rest mass m = 0 (photon) are related by

$$E = pc$$

each with magnitude of momentum

$$p_\gamma = \frac{E_\gamma}{c} = 1.011 \text{ MeV/c}$$

The momentum vectors of the photons make angles $\pm\theta$ with the x-axis. Conservation of momentum in the x-direction is

$$p = 2p_\gamma \cos\theta$$

Hence,

$$\theta = \cos^{-1}\left(\frac{p}{2p_\gamma}\right) = \cos^{-1}\left(\frac{1.422}{2 * 1.011}\right) = 45.3°$$

Answer: $E_\gamma = 1.011$ MeV; $p_\gamma = 1.011$ MeV/c; $\theta = 45.3°$.