

Answer on Question #51217, Physics, Other

A muon has a lifetime of 2×10^{-6} s in its rest frame. It is created 100 km above the earth and moves towards it at a speed of 2.97×10^8 ms $^{-1}$. At what altitude does it decay? What is the distance travelled by the muon during this time in its own frame of reference?

Solution:

The time dilation factor is

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and the muon's lifetime τ in its rest frame corresponds to $\gamma\tau$ in the laboratory frame. This means the muon travels a distance $v\gamma\tau$, or

$$d = v\gamma\tau = \frac{2.97 \times 10^8 \times 2 \times 10^{-6}}{\sqrt{1 - \left(\frac{2.97 \times 10^8}{3 \times 10^8}\right)^2}} = 4210.7 \text{ m} \approx 4.21 \text{ km}$$

before it decays, which occurs at 95.8 km above the ground.

According to the muon in its own frame of reference, it has only traveled

$$\frac{d}{\gamma} = v\tau = 2.97 \times 10^8 \times 2 \times 10^{-6} = 594 \text{ m} \approx 0.6 \text{ km}$$