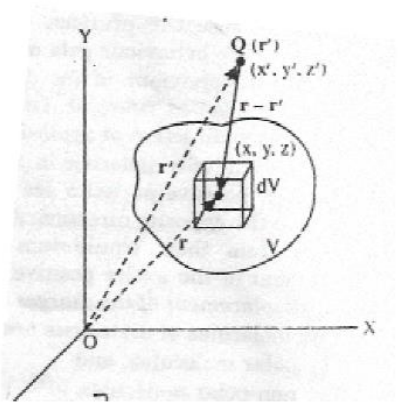
Answer on Question #51175-Physics-Electromagnetism

A dielectric object is placed in an electric field. The object becomes polarized and a large number of atomic /molecular dipoles in the object align in the direction of the applied electric field. Derive an expression for the electric field produced by this polarized dielectric at a point outside the dielectric object.

Solution

In the presence of an electric field, a dielectric becomes polarized and acquires a dipole moment P, per unit volume. The aim is to calculate field at point r [denoted by Q(r') in Fig. outside the dielectric. Let the volume of the dielectric is divided into small elementary volumes and



consider one such volume element dV of the dielectric. The potential due to dielectric body occupying volume dV as observed outside dV for single dipole potential can be expressed as,

$$\phi(r') = \frac{1}{4\pi\varepsilon_0}\int_V \frac{P(r).(r-r')}{|(r-r')|^3}dV$$

$$\phi(r') = \frac{1}{4\pi\varepsilon_0}\int_V \frac{P(r).\frac{(r-r')}{|(r-r')|}dV}{|(r-r')|^2}$$

$$\phi(r') = \frac{1}{4\pi\varepsilon_0}\int_V \frac{P(r).(r-r')^0}{|(r-r')|^2}dV$$

where (r-r')0 represents unit vector directed from Q towards dV. It is obviously equal to

$$\frac{(x'-x)i + (y'-y)j + (z'-z)k}{\sqrt{\{(x'-x)^2 + (y'-y)^2 + (z'-z)^2\}}}$$

where x', y',z' are the Cartesian coordinates of point r' and (x,y,z)those at r. Further,

$$(r-r')^0 = -\left[i\frac{\partial}{\partial x}(r-r') + j\frac{\partial}{\partial x}(r-r') + k\frac{\partial}{\partial x}(r-r')\right]$$

$$P(r) = iP_x(x.y.z) + jP_y(x.y.z) + kP_z(x.y.z)$$

substituing these values in equation, we get

$$\phi(r') = \frac{1}{4\pi\varepsilon_0}\left\{\int_V \frac{P_x}{(r-r')^2}.\frac{\partial}{\partial x}(r-r')dV + \int_V \frac{P_y}{(r-r')^2}.\frac{\partial}{\partial y}(r-r')dV + \int_V \frac{P_z}{(r-r')^2}.\frac{\partial}{\partial z}(r-r')dV\right\}$$

$$= \phi_1 + \phi_2 + \phi_3$$

Now we shall consider the integrals separately. It is obvious that

$$-\frac{1}{|(r-r')|^2}.\frac{\partial(r-r')}{\partial x} = \frac{\partial}{\partial x}\frac{1}{|(r-r')|^2}$$

$$dV = dxdydz$$

Therefore

$$\phi_1 = \frac{1}{4\pi\varepsilon_0}\iiint P_x \frac{\partial}{\partial x}\frac{1}{|(r-r')|^2}dxdydz$$

After integration we get

$$\phi(r') = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\mathbf{P}.n}{|(r-r')|}dA - \frac{1}{4\pi\varepsilon_0}\int_V \frac{(-divP)dV}{|(r-r')|}$$

Equation obviously shows that the potential at point r' due to dielectric body is the same as if it were replaced by a system of bound charges in empty space. A part of these bound charges appears on the dielectric surfaces as a surface density óP' given by

$$\sigma_P' = \mathbf{P.n}$$

while the remaining bound charge appears throughout the volume V as a volume density rp' given as

$$\rho_P' = -\left(\frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}\right)$$

The intensity of electric field due to the polarized volume may be expressed as

$$E(r') = \frac{1}{4\pi\varepsilon_0}\left[\int_S \frac{\sigma_P'(r-r')}{|r-r'|^3}dA + \int_V \frac{\rho_P'(r-r')}{|r-r'|^3}dV\right]$$