Answer on Question #51127-Physics- Electrodynamics

Do the following fields satisfy all four Maxwell's equations?

$$\overrightarrow{E(t)} = \overrightarrow{E_0} \sin x \sin t \,.\, \vec{B} = \overrightarrow{B_0} \cos x \cos t \,.$$

Solution

1) $\nabla \cdot \vec{B} = \frac{\partial}{\partial x}(B_{0x} \cos x \cos t) = B_{0x} \cos t \frac{\partial}{\partial x}(\cos x) = -B_{0x} \sin x \cos t = 0 \; \forall t \; only \; if \; B_{0x} = 0.$

2) $\nabla \times \overrightarrow{E(t)} = \vec{k} \, \frac{\partial}{\partial x}\left(E_{0y} \sin x \sin t\right) - \vec{j} \frac{\partial}{\partial x}(E_{0z} \sin x \sin t) = \left(0; -E_{0z} \cos x \sin t \,; E_{0y} \cos x \sin t\right).$

$$\frac{\partial \vec{B}}{\partial t} = -\overrightarrow{B_0} \cos x \sin t.$$

Thus

$$\nabla \times \overrightarrow{E(t)} = -\frac{\partial \vec{B}}{\partial t}\frac{1}{c} \; only \; if \; B_{0y} = -cE_{0z} \; and \; B_{0z} = cE_{0y}.$$

3) $\nabla \cdot \vec{E} = \frac{\partial}{\partial x}(E_{0x} \sin x \sin t) = E_{0x} \cos t \frac{\partial}{\partial x}(\sin x) = E_{0x} \cos x \cos t = 0 \; \forall t \; only \; if \; E_{0x} = 0.$

4) $\frac{\partial \vec{E}}{\partial t} = \overrightarrow{E_0} \sin x \cos t.$

$$\nabla \times \overrightarrow{B(t)} = \vec{k} \, \frac{\partial}{\partial x}\left(B_{0y} \cos x \cos t\right) - \vec{j} \frac{\partial}{\partial x}(B_{0z} \cos x \cos t) = \left(0; B_{0z} \sin x \cos t \,; -B_{0y} \sin x \cos t\right).$$

So,

$$\nabla \times \overrightarrow{B(t)} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \; only \; if \; cB_{0y} = -E_{0z} \; and \; cB_{0z} = E_{0y}.$$

So, $cB_{0y} = \frac{1}{c} B_{0y} \to c = \frac{1}{c}$. This is not true. Thus $\overrightarrow{E_0} = \overrightarrow{B_0} = \vec{0}$.

Therefore the following fields don't satisfy all four Maxwell's equations.

(If we work in system where $c = \frac{1}{c} = 1$ the following fields satisfy all four Maxwell's equations only if

$\overrightarrow{E_0} = (0; E; -B) \; and \; \overrightarrow{B_0} = (0; B; E)$, where E and B are arbitrary constants.)