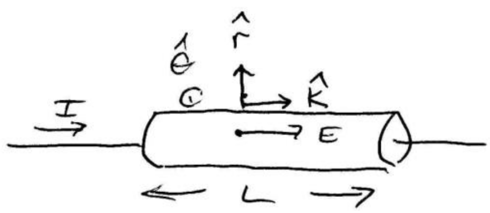
Answer on Question #51124-Physics-Electrodynamics

Determine the magnitude of the pointing vector and the energy per unit time delivered to a wire of length L and cross-section A when electric current I flows in it.

Solution

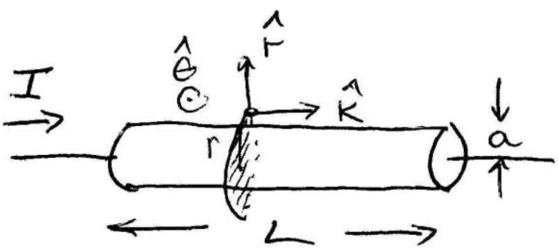
We choose unit vectors as shown in the figure below



There is an electric potential difference across the resistor that is equal to $\Delta V = IR$. The electric field is uniform in the resistor so $|\Delta V| = \left|\int \vec{E} d\vec{s}\right| = EL = IR$. Therefore the direction and magnitude of the electric field is given by

$$\vec{E} = \frac{IR}{L}\hat{k}.$$

We can use Ampere's Law to find the magnetic field surface of the resistor. Choose a circular loop as shown in the figure of radius $= a$.



Ampere's law is

$$\oint_{circle} \vec{B} d\vec{s} = \mu_0 \iint_{disk} \vec{J}\vec{n}\, da.$$

By symmetry,

$$\oint_{circle} \vec{B} d\vec{s} = B2\pi a.$$

The current through the disk is

$$\iint_{disk} \vec{J}\vec{n}\, da = I.$$

So Ampere's Law becomes

$$B2\pi a = \mu_0 I.$$

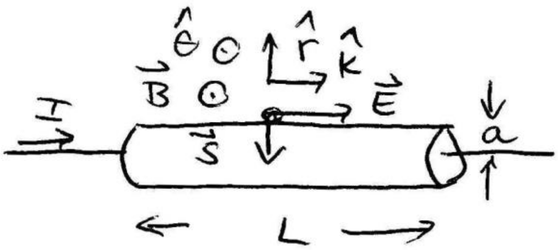
With our choice of unit vectors, the direction and magnitude of the magnetic field on the surface of the resistor is

$$\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{\theta}.$$

The **Poynting vector** is given by

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{\left(\frac{IR}{L}\hat{k}\right)\times\left(\frac{\mu_0 I}{2\pi a}\hat{\theta}\right)}{\mu_0} = \frac{IR}{L}\frac{I}{2\pi a}(-\hat{r})$$

In order to calculate the power flow into the resistor we need to calculate the closed surface flux of the Poytning vector over the surface of the resistor.



Because the Poytning vector points radially inward we only calculate the flux over the cylindrical body of the resistor. Hence we set $r = a$ for Poynting vector and then integrate it over the outside cylindrical area of the resistor. For a closed surface we choose $\hat{n}_{out} = \hat{r}$ and so $\vec{S}\hat{r} = -|\vec{S}|$.

$$Power = \oiint \vec{S}\,\hat{n}_{out} da = \iint_{cylinder} \vec{S}\hat{r}\, da = -\iint_{cylinder} |\vec{S}|\, da = -|\vec{S}(r = a)|A.$$

The surface area in question is $= 2\pi a L$. Thus using our result for the magnitude of the Poynting vector we have that the power flow through the surface is

$$Power = -\left(\frac{IR}{L}\frac{I}{2\pi a}\right)2\pi a L = -I^2 R.$$

The negative sign indicates that power is flowing into the resistor. We know that the rate that work is done by the electric field in the wire is I^2R so our answer makes because energy per sec flows in and is equal to the rate that work is done on the charges in the wire. This energy is dissipated in the form of random thermal motion due to collisions in the wire and eventually is transmitted to the motion of the air molecules surrounding the wire.

The energy per unit time delivered to a wire is

$$P = I^2 R.$$