Answer on Question 51123, Physics, Electric Circuits

Question:

In a cyclotron, the diameter of the pole faces is $100cm$ and the magnetic field between the pole faces is $0.60T$. The cyclotron is used for accelerating protons. Calculate the kinetic energy of proton in eV and speed of the proton as it emerges from the cyclotron. Also determine the cyclotron frequency.

Solution:

There are two forces that acts on the proton when it moves in the uniform magnetic field: the magnetic force and the radial force (this one is required to keep the proton moving in a circle). So, using Newton's second law of motion we can write:

$$qvB = \frac{mv^2}{r},$$

where, $q = +1e = 1.6 \cdot 10^{-19}C$ is the charge of the proton, v is the speed of the proton, B is the magnetic field, $m = 1.67 \cdot 10^{-27}kg$ is the mass of the proton, r is the radius of the proton's path.

From this formula we can easily find the speed of the proton, and as we know the speed, we can obtain the kinetic energy of the proton:

$$v = \frac{qBr}{m},$$

$$KE = \frac{1}{2}mv^2 = \frac{m}{2}\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m} = \frac{(1.6 \cdot 10^{-19}C)^2 \cdot (0.6T)^2 \cdot (0.5m)^2}{2 \cdot 1.67 \cdot 10^{-27}kg} =$$
$$= 6.898 \cdot 10^{-13}J = 6.898 \cdot 10^{-13} \cdot 6.24 \cdot 10^{18}eV = 4.3MeV.$$

So, let us return to the formula for the speed of the proton and obtain it:

$$v = \frac{qBr}{m} = \frac{1.6 \cdot 10^{-19}C \cdot 0.6T \cdot 0.5m}{1.67 \cdot 10^{-27}kg} = 2.87 \cdot 10^7 \frac{m}{s}.$$

We know that $f = 1/T$, where f is the cyclotron frequency, and $T = \frac{2\pi r}{v}$ is the period of the motion that is equal to the circumference of the circle divided by the speed of the proton. So, after substituting into this formula the radius of the proton's path that we can easily obtain from the equation for the Newton's second law of motion, we finally get the cyclotron frequency:

$$f = \frac{1}{T} = \frac{1}{\frac{2\pi \frac{mv}{qB}}{v}} = \frac{qB}{2\pi m} = \frac{1.6 \cdot 10^{-19} C \cdot 0.6T}{2 \cdot 3.14 \cdot 1.67 \cdot 10^{-27} kg} = 9.15 MHz.$$

Answer:

$KE = 4.3 MeV.$

$v = 2.87 \cdot 10^{7} \frac{m}{s}.$

$f = 9.15 MHz.$