Answer on Question#51120 - Physics - Electric Circuits

A dielectric object is placed in an electric field. The object becomes polarised and a large number of atomic/molecular dipoles in the object align in the direction of the applied electric field. Derive an expression for the electric field produced by this polarised dielectric at a point outside the dielectric object.

Solution:

The electric field in which this object is placed, defines the polarization $\boldsymbol{P}(\boldsymbol{r})$ of this object ($\boldsymbol{r}$ – radius-vector inside this object). The polarization depends on the form of external field and on properties of the dielectric object. To derive the expression for the electric field produced by this polarization let's first consider an infinitesimal volume dV of the dielectric at the point with radius-vector $\boldsymbol{r}'$. It can be considered as an electric dipole with a dipole moment $\boldsymbol{p}(\boldsymbol{r}') = \boldsymbol{P}(\boldsymbol{r}')dV$. It is known that the electric field at the point with radius-vector $\boldsymbol{R}$ produced by the electric dipole at the origin with dipole moment $\boldsymbol{p}$ is given by

$$\boldsymbol{E}(\boldsymbol{R}) = \frac{3(\boldsymbol{p}\cdot\boldsymbol{R})}{R^5}\boldsymbol{R} - \frac{\boldsymbol{p}}{R^3}$$

Therefore, the electric field at the point with radius-vector $\boldsymbol{r}$ produced by the infinitesimal volume at the point with radius-vector $\boldsymbol{r}'$ takes the following form

$$d\boldsymbol{E}(\boldsymbol{r}) = \left[\frac{3\big(\boldsymbol{P}(\boldsymbol{r}')\cdot(\boldsymbol{r}-\boldsymbol{r}')\big)}{|\boldsymbol{r}-\boldsymbol{r}'|^5}(\boldsymbol{r}-\boldsymbol{r}') - \frac{\boldsymbol{P}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\right]dV$$

The electric field produced by the whole object is defined the sum of all electric fields produced by such infinitesimal volumes of the object, in other words it defines by integrating the previous equation over the volume of the object:

$$\boldsymbol{E}(\boldsymbol{r}) = \int\limits_V \left[\frac{3\big(\boldsymbol{P}(\boldsymbol{r}')\cdot(\boldsymbol{r}-\boldsymbol{r}')\big)}{|\boldsymbol{r}-\boldsymbol{r}'|^5}(\boldsymbol{r}-\boldsymbol{r}') - \frac{\boldsymbol{P}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\right]dV$$

Answer: $\boldsymbol{E}(\boldsymbol{r}) = \int_V \left[\frac{3\big(\boldsymbol{P}(\boldsymbol{r}')\cdot(\boldsymbol{r}-\boldsymbol{r}')\big)}{|\boldsymbol{r}-\boldsymbol{r}'|^5}(\boldsymbol{r}-\boldsymbol{r}') - \frac{\boldsymbol{P}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\right]dV.$