

Answer on Question#51119 - Physics - Electric Circuits

A sphere of radius R carries a charge of volume charge density $\rho(r) = cr$, where c is a constant and r denotes the distance from the centre of the sphere. Calculate the total charge enclosed by the sphere and the electric field at points lying inside and outside the sphere.

Solution:

The total charge of the sphere is obtained by integrating the charge density over the inner volume of the sphere.

$$Q = \int \rho(r) dV$$

Due to spherical symmetry, dV can be written in the following way

$$dV = 4\pi r^2 dr$$

Therefore,

$$Q = \int_0^R \rho(r) 4\pi r^2 dr = c \int_0^R 4\pi r^3 dr = \pi c R^4$$

The electric field at the point lying inside the sphere is defined only by charges that lie closer to the centre than this point. Therefore, the electric field at the distance r from the centre (inside the sphere) is defined by the charge which lies inside the sphere of radius r ($Q(r) = \pi c r^4$) and it's given by

$$E_{in}(r) = \frac{Q(r)}{r^2} = \pi c r^2$$

The electric field outside the sphere is defined by the whole charge of the sphere and its value at the distance r ($r > R$) from the centre is given by

$$E_{out}(r) = \frac{Q}{r^2} = \pi c \frac{R^4}{r^2}$$

Answer: $Q = \pi c R^4$;

$$E(r) = \begin{cases} \pi c r^2, & r \leq R \\ \pi c \frac{R^4}{r^2}, & r > R \end{cases}$$