Answer on Question #51096-Physics-Other

When meteors enter the atmosphere, they are typically traveling at around 15km/s. At such speeds, heat does not have time to be conducted away and even the air has no time to be pushed out of the way. This results in adiabatic compression of the air ahead of the meteor. The density of the atmosphere in kg/m3 is given by

$$\rho = 1.22e^{-\frac{h}{8100}}$$

where h is the altitude in meters.

a) For a spherical meteor of radius 5m entering the atmosphere on a vertical trajectory, As a result of compressing all the air in its way into a cylinder of radius $r = 5.5m$ and height $h_1 = 0.3m$, there will be a very high pressure and temperature in front of the meteor. Calculate the density of air in this compression zone when the meteor has reached an altitude of $h_0 = 4050m$ by finding the kg of air compressed and the volume of the compression zone. Hint: $dm = \rho A dh$

b) Calculate the original volume of the gas before it was compressed. You may use $H = 40000m$ as the top of the atmosphere.

Solution

a) The mass of air being compressed is

$$m = \int_{h_0}^{H} \rho A dh = A \int_{h_0}^{H} 1.22e^{-\frac{h}{8100}} dh$$

$$= 1.22 \cdot 8100 \cdot \pi r^2 \int_{\frac{h_0}{8100}}^{\frac{(H)}{8100}} e^{-\frac{h}{8100}} d\left(\frac{h}{8100}\right) = 1.22 \cdot 8100 \cdot \pi 5.5^2 \left(e^{-\frac{h_0}{8100}} - e^{-\frac{H}{8100}}\right)$$

$$= 5.6 \cdot 10^5 kg.$$

The volume of the compression zone is

$$V = Ah_1 = \pi r^2 h_1 = \pi 5.5^2 \cdot 0.3 = 28.5\ m^3.$$

The density of air in this compression zone is

$$\rho_{h_0} = \frac{m}{V} = \frac{5.6 \cdot 10^5 kg}{28.5\ m^3} = 2.0 \cdot 10^4 \frac{kg}{m^3}.$$

b) The original volume of the gas before it was compressed is

$$V_0 = A(H - h_0) = \pi r^2 (H - h_0) = \pi 5.5^2 (40000 - 4050) = 3.4 \cdot 10^6 m^3.$$