Answer on Question #51032, Physics, Quantum Mechanic

Find the average value of P_X. $\langle P_X \rangle$ for n=1 state of a particle in a one dimensional box of length L. Comment on your result.

Solution

The wave function of a one-dimensional potential well is given by Eq.(1)

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right) \tag{1}$$

where L is length of the box.

The momentum operator

$$\hat{P}_X = -i\hbar \frac{\partial}{\partial x} \tag{2}$$

Find the average value of

$$\langle P_X \rangle = \int_0^L \psi_n^*(x) \hat{P}_X \psi_n(x)\, dx = \int_0^L \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right)\left(-i\hbar \frac{\partial}{\partial x}\right)\sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right) dx =$$

$$-\frac{2i\hbar}{L}\int_0^L \sin\left(\frac{\pi n x}{L}\right)\left(\frac{\partial}{\partial x}\right)\sin\left(\frac{\pi n x}{L}\right) dx = -\frac{2i\hbar}{L}\frac{\pi n}{L}\int_0^L \sin\left(\frac{\pi n x}{L}\right)\cos\left(\frac{\pi n x}{L}\right) dx =$$

$$-\frac{i\hbar \pi n}{L^2}\int_0^L \sin\left(\frac{2\pi n x}{L}\right) dx = -\frac{i\hbar \pi n}{L^2}\cdot\frac{L}{2\pi n}\cos\left(\frac{2\pi n x}{L}\right)\Bigg|_0^L = -\frac{i\hbar}{2L}\left(\cos\left(\frac{2\pi n L}{L}\right) - cos 0\right) =$$

$$= -\frac{i\hbar}{2L}(1-1) = 0$$

The projection of the momentum is the value fluctuates near the equilibrium position, like a pendulum, and varying in magnitude and direction. Therefore, the average value of the projection of the momentum is zero.