Answer on Question #50966-Physics-Other

A uniform ball, hoop and disk, all of mass M=6 kg and radius R roll smoothly from rest down a ramp inclined at 30 to the horizontal. Which of the three objects reaches the bottom of the slope first?

Solution

The moments of inertia for a sphere $=\frac{2}{5}MR^2$; disk $=\frac{1}{2}MR^2$; and a hoop $=MR^2$.

The fraction of kinetic energy which goes into translational motion,

$$f = \frac{\frac{1}{2}Mv_{com}^{2}}{\frac{1}{2}Mv_{com}^{2} + \frac{1}{2}I_{com}\omega^{2}}.$$

In general, $I_{com} = \beta M R^2$, where β is a constant and $\omega = \frac{v_{com}}{R}$.

$$f = \frac{\frac{1}{2}Mv_{com}^2}{\frac{1}{2}Mv_{com}^2 + \frac{1}{2}\beta MR^2 \left(\frac{v_{com}}{R}\right)^2} = \frac{1}{1+\beta}.$$

For hoop, $\beta = 1, f = 0.5$. For disk, $\beta = \frac{1}{2}, f = 0.67$. For hoop, $\beta = \frac{2}{5}, f = 0.71$.

Sphere rolls fastest, followed by the disk. The last will come a hoop.

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