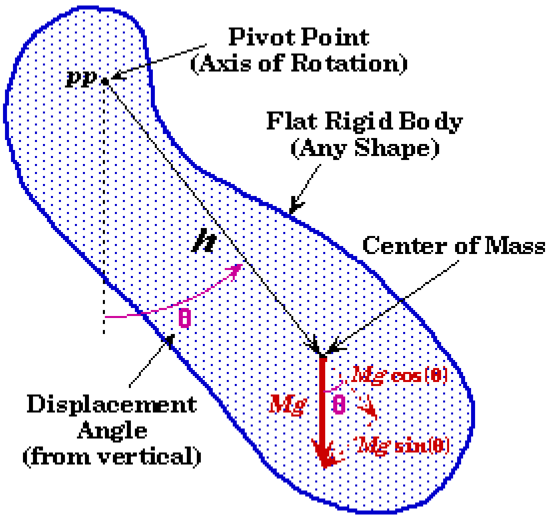
Answer on Question #50891, Physics, Mechanics | Kinematics | Dynamics

Define and explain physical pendulum.

Answer:

If a flat, rigid body is pivoted about any point other than its center of mass and displaced by a small angle, the body will execute SHM (Simple Harmonic Motion).



The derivation is similar to that of a simple pendulum since one can consider all the mass **M** to be located at the body's center of mass. Then a physical pendulum looks like a simple pendulum except that its moment of inertia is found using the parallel axis theorem.

$$I_{pp} = I_{cm} + M h^2$$

I_{cm} = the body's moment of inertia about its center of mass.

h = Distance from pivot point to the center of mass.

M = Mass of the body.

If the pivot joint is frictionless then the net torque acting on the planar object is given by the force of gravity perpendicular to lever arm, **Mg sin(q)**, times the length of the lever arm, **h:**

$$\tau_{net} = -M g \sin(\theta) h = I_{pp} \alpha$$

When the angle of oscillation is small then the value of sin(q) and q or nearly the same provided q is measured in radians. Using this approximation, the above torque equation can be solved for the angular acceleration,

$$\alpha = -\frac{M g h}{I_{pp}} \theta$$

Since a =**d^2**q/**dt^2**, this equation is structurally similar to the differential equation for any type of SHM,

$$\frac{d^2\theta}{dt^2} = -\frac{M g h}{I_{pp}}\theta$$

Matching terms with SHM equations,

$$\omega = \sqrt{\frac{Mg h}{I_{pp}}} = \sqrt{\frac{Mg h}{I_{cm} + M h^2}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_{cm} + M h^2}{Mg h}}$$