Answer on Question #50890-Physics-Mechanics-Kinematics-Dynamics

Let a copper rod of mass m rest on two horizontal rails that are L meters apart and carry a current of I amperes from one rail to another. Let the coefficient of static friction between the rod and the rails be u. What is the (1) magnitude and (2) angle (with respect to the vertical) of the smallest magnetic field that puts the rod on verge of sliding?

Solution

The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; mg, the magnitude of the (downward) force of gravity; $\vec{F}_N$, the normal force exerted by the stationary rails upward on the rod; and $\overrightarrow{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that $\overrightarrow{f}$ points westward (and is equal to its maximum possible value uF_N). Thus, $\vec{F}$ has an eastward component F_x and an upward component F_y, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the right-hand rule, a downward component (B_d) of $\vec{B}$ will produce the eastward F_x, and a westward component (B_w) will produce the upward F_y. Specifically,

$$F_x = ILB_d, F_y = ILB_w.$$

Considering forces along a vertical axis, we find

$$F_N = mg - F_y = mg - ILB_w$$

so that

$$f = f_{max} = u(mg - ILB_w).$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x - f = 0 \rightarrow ILB_d = u(mg - ILB_w).$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$IBL\cos\theta = u(mg - ILB\sin\theta) \rightarrow B = \frac{umg}{IL(\cos\theta + u\sin\theta)}.$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1} u.$$

Answer: (1) $\frac{umg}{IL(\cos\theta + u\sin\theta)}$; **(2)** $\tan^{-1} u$.