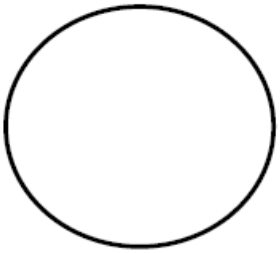
Answer on Question #50854- Physics-Electromagnetism

A time varying magnetic field $\overrightarrow{B(t)} = \overrightarrow{B_0}\, coswt$ pointing out of the page fills the region enclosed by a circle of radius a shown in the figure below. Determine the induced electric field.

Solution

According to Lenz's law, the direction of $\overrightarrow{E_{ind}}$ must be such that it would drive the induced current to produce a magnetic field opposing the change in magnetic flux. With the area vector $\vec{A}$ pointing in the page, the magnetic flux is negative or inward. In the region $r < a$, the rate of change of magnetic flux is



$$\frac{d\Phi}{dt} = \frac{d}{dt}\left(\vec{B}\cdot\vec{A}\right) = \frac{d}{dt}(-AB) = -\frac{dB}{dt}\pi r^2.$$

$$\oint \overrightarrow{E_{ind}}\, d\vec{s} = E_{ind}\cdot 2\pi r = -\frac{d\Phi}{dt} = \frac{dB}{dt}\pi r^2.$$

which implies

$$E_{ind} = \frac{r}{2}\frac{dB}{dt} = -\omega\frac{r}{2}B_0 \sin\omega t.$$

Similarly, for $r > a$, the induced electric field may be obtained as

$$E_{ind}\cdot 2\pi r = -\frac{d\Phi}{dt} = \frac{dB}{dt}\pi a^2.$$

$$E_{ind} = \frac{a^2}{2r}\frac{dB}{dt} = -\omega\frac{a^2}{2r}B_0 \sin\omega t.$$