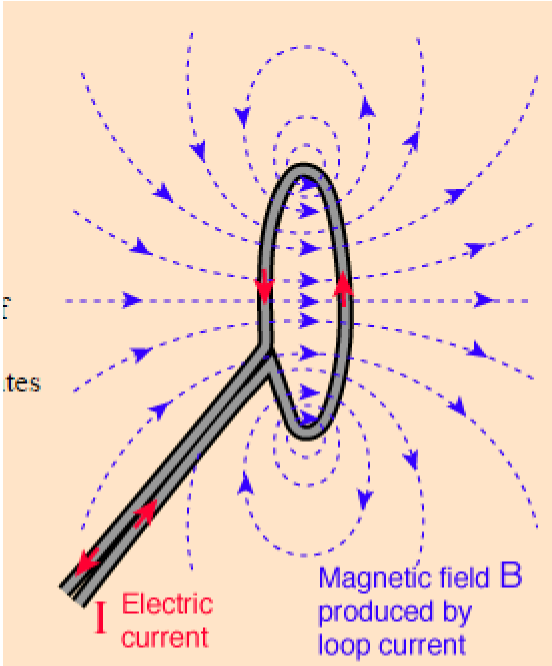
Answer on Question #50851, Physics, Electromagnetism

Using Biot-Savart Law, obtain an expression for the magnetic field along the axis of a current loop.

ANSWER:
Electric current in a circular loop creates a magnetic field which is more concentrated in the center of the loop than outside the loop. Stacking multiple loops concentrates the field even more into what is called a solenoid.

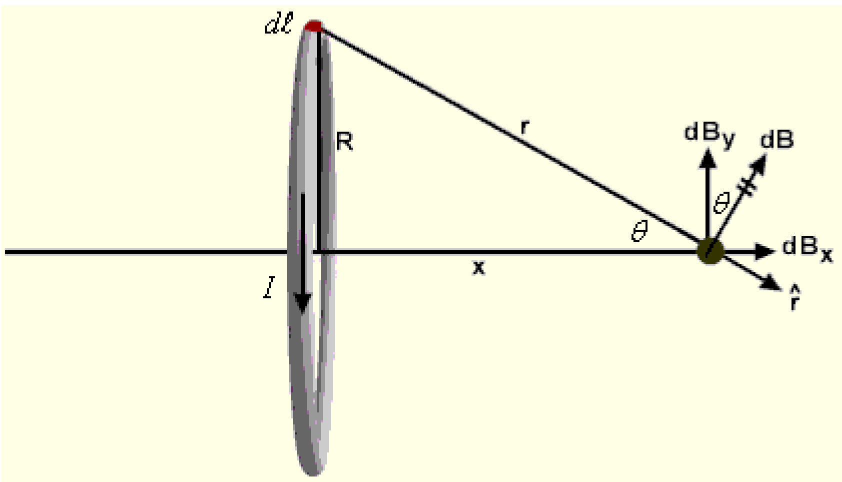


Let's begin with a basic statement of the Biot-Savart Law.

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl \times \boldsymbol{r}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin(90°)}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$



As shown in the animation, the components perpendicular to the loop's axis, dB_y, will cancel as we integrate around the loop. Thus, we will focus only the horizontal components, dB_x.

$$dB_x = dB\,\sin(\theta)$$
$$dB_x = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\sin(\theta)$$

$$dB_x = \frac{\mu_0}{4\pi}\frac{I\,R}{r^3}dl$$

Using the Pythagorean Theorem, we can express r in terms of x and R,

$$r = \sqrt{x^2 + R^2}$$

Giving as

$$dB_x = \frac{\mu_0}{4\pi}\frac{I\,R}{\sqrt{x^2 + R^2}^3}dl$$

Our last step is to calculate the resultant magnetic field by adding up all of these contributions.

$$B_x = \oint dB_x$$
$$B_x = \oint \frac{\mu_0}{4\pi}\frac{I\,R}{\sqrt{x^2 + R^2}^3}dl$$
$$B_x = \frac{\mu_0}{4\pi}\frac{I\,R}{\sqrt{x^2 + R^2}^3}\oint dl$$

$$B_x = \frac{\mu_0}{4\pi}\frac{I\,R}{\sqrt{x^2 + R^2}^3}2\pi R$$

$$B_x = \frac{\mu_0}{2}\frac{I\,R^2}{\sqrt{x^2 + R^2}^3}$$