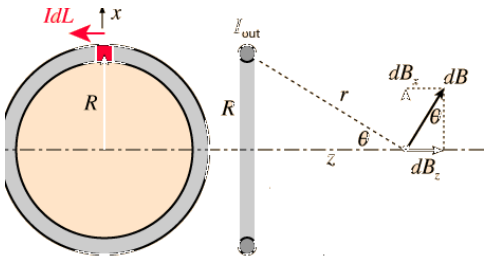
Answer on Question #50821-Physics-Field Theory

A) Using Biot-Savart Law, obtain an expression for the magnetic field along the axis of a current loop.

B) In the Bohr model of hydrogen atom, the electron follows a circular orbit centered on the nucleus containing a proton. The motion of the electron along the circular orbit constitutes a current. Calculate the magnetic field produced by the orbiting electron at the site of the proton.

C) A fuse in an electric circuit is designed to open the circuit like a switch when the current exceeds a preset value. If a fuse is made of material that melts when the current density reaches $j = 400\,\frac{A}{cm^2}$, what is the diameter of the wire d needed to limit the current to $I_{lim} = 0.30\ A$?

Solution



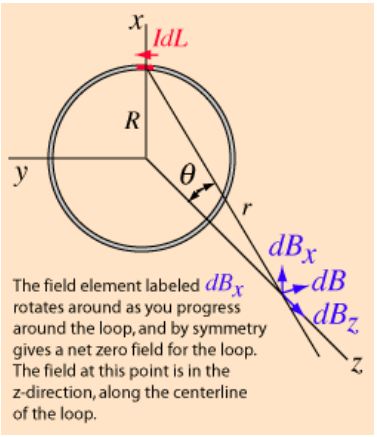
A) $dB = \frac{\mu_0}{4\pi}\frac{IdL}{r^2}$

$r^2 = z^2 + R^2.$

Substitutiing $dB_z = dB\sin\theta$ and $\sin\theta = \frac{R}{\sqrt{z^2+R^2}}$ gives

$$dB_z = \frac{\mu_0 IdL}{4\pi}\frac{R}{(z^2+R^2)^{\frac{3}{2}}}.$$

The application of the Biot-Savart law on the centerline of a current loop involves integrating the z-component.



The symmetry is such that all the terms in this element are constant except the distance element dL , which when integrated just gives the circumference of the circle. The magnetic field is then

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(z^2+R^2)^{\frac{3}{2}}}.$$

B) An expression for the magnetic field at the center of a current loop is

$$B = \frac{\mu_0}{2}\frac{I}{R},$$

where R is a raius of orbit. A current is

$$I = \frac{e}{T} = ef.$$

So,

$$B = \frac{\mu_0}{2}\frac{ef}{R}.$$

Frequncy of electron is

$$f_n = \frac{n\hbar}{mR_n}.$$

Thus

$$B = \frac{\mu_0}{2}\frac{en\hbar}{mR_n^2}.$$

For $n = 1$:

$$B = \frac{\mu_0}{2}\frac{\left(\frac{e}{m}\right)\hbar}{a_0^2}.$$

a_0 is Bohr radius.

$$B = \frac{4\pi \cdot 10^{-7}}{2}\frac{1.759 \cdot 10^{11} \cdot 1.054 \cdot 10^{-34}}{(0.059 \cdot 10^{-9})^2} = 3.35 \cdot 10^{-9} T.$$

C) The current density of the wire is

$$j = \frac{I}{A} = \frac{I}{\frac{\pi d^2}{4}}.$$

Thus,

$$d_{lim} = 2\sqrt{\frac{I}{\pi j}} = 2\sqrt{\frac{0.30\ A}{\pi \cdot 400\frac{A}{cm^2}}} = 0.015\ cm = 0.15\ mm.$$