50744, Physics, Optics

Question Show that: 1) $u = e^x \sin y$ is a solution of Laplace's equation. 2) $u = x^2 + t^2$ is a solution of the wave equation.

Solution 1.Laplace equation is

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

Let us substitute given function. We get

$$\frac{\partial^2 (e^x \sin y)}{\partial x^2} + \frac{\partial^2 (e^x \sin y)}{\partial y^2} = e^x \sin y + \frac{\partial (e^x \cos y)}{\partial y} = e^x \sin y - e^x \sin y = 0$$

So, it satisfies Laplace equation.
2. Wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Let us substitute given function. We get

$$\frac{\partial^2 (x^2 + t^2)}{\partial t^2} - c^2 \frac{\partial^2 (x^2 + t^2)}{\partial x^2} =$$

$$= 2\frac{\partial t}{\partial t} - c^2 \frac{\partial x}{\partial x} = 1 - c^2 = 0$$

So, it satisfies wave equation, if $c^2 = 1$.