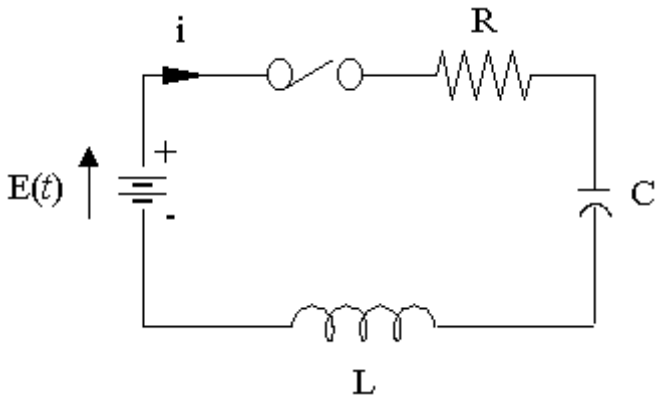
Answer on Question #50741, Physics, Other

A series circuit consists of a resistor with R = 20ohm, an inductor with L = 1H, a capacitor with C = 0.002 F, and a 12 V battery. IF the initial charge and current are both 0, find the charge and current at time t.

Solution:



Consider a series RLC circuit (one that has a resistor, an inductor and a capacitor) with a constant driving electro-motive force (emf) E. The current equation for the circuit is

$$L\frac{di}{dt} + Ri + \frac{Q}{C} = E$$

Since , this equation becomes

$$L\frac{d^2Q}{dt^2} + \frac{RdQ}{dt} + \frac{Q}{C} = E$$

With the given values of R, L, C and E, the last equation becomes

$$\frac{d^2Q}{dt^2} + 20\frac{dQ}{dt} + \frac{Q}{0.002} = 12$$

$$\frac{d^2Q}{dt^2} + 20\frac{dQ}{dt} + 500Q = 12$$

The auxiliary equation is

$$r^2 + 20r + 500 = 0$$

with roots

$$r = -10 - 20\,i$$
$$r = -10 + 20\,i$$

so the solution of the complementary equation is

$$Q_c(t) = e^{-10t}(c_1 \cos 20t + c_2 \sin 20t)$$

For the method of undetermined coefficients we try the particular solution

$$Q_p(t) = A$$
$$500A = 12$$

or $A = \frac{3}{125} = 0.024$

The general solution is

$$Q(t) = Q_c(t) + Q_p(t) = e^{-10t}(c_1 \cos 20t + c_2 \sin 20t) + 0.024$$

But

$$Q(0) = 0 = c_1 + 0.024$$

$$c_1 = -0.024$$

The current

$$I(t) = Q'(t) = e^{-10t}[(-10c_1 + 20c_2)\cos 20t + (-10c_2 - 20c_1)\sin 20t]$$

$$I(0) = 0 = -10c_1 + 20c_2$$

Thus

$$c_2 = \frac{c_1}{2} = -0.012$$

Hence the charge is

$$Q(t) = e^{-10t}(-0.024\cos 20t - 0.012\sin 20t) + 0.024 =$$
$$= -0.012e^{-10t}(2\cos 20t + \sin 20t) + 0.024$$

The current is

$$I(t) = Q'(t) = e^{-10t}[(10 * 0.024 - 20 * 0.012)\cos 20t + (10 * 0.012 + 20 * 0.024)\sin 20t] =$$
$$= e^{-10t}(0.6)\sin 20t$$

Answer: $Q(t) = -0.012e^{-10t}(2\cos 20t + \sin 20t) + 0.024$

$I(t) = 0.6e^{-10t}\sin 20t$