

Answer on Question #50740-Physics-Other

The rate at which a spherical pill (tablet) dissolves in the body after being consumed is described by the rate of change of its volume with time $\frac{dV}{dt}$. Given that the rate of change of volume is directly proportional to its surface area, compute the time in which 80% of a pill of size 4 cm would dissolve, given that it dissolves completely in 2 hrs.

Solution

$$\frac{dV}{dt} = -kA.$$

$$A = 4\pi r^2, \quad V = \frac{4}{3}\pi r^3.$$

$$\frac{dV}{dt} = \frac{d}{dt} \left(\frac{4}{3}\pi r^3 \right) = \frac{d}{dr} \left(\frac{4}{3}\pi r^3 \right) \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}.$$

$$\text{So, } k = -\frac{dr}{dt} > 0.$$

We know,

$$\frac{dr}{dt} = -k \rightarrow r = r_0 - kt.$$

Thus,

$$0 = 4cm - k \cdot 2 \text{ hrs} \rightarrow k = 2 \frac{\text{cm}}{\text{hrs}}.$$

$$\frac{V(T)}{V_0} = (1 - 0.8) = 0.2 \rightarrow \frac{\left(\frac{4}{3}\pi r(T)^3\right)}{\left(\frac{4}{3}\pi r_0^3\right)} = \left(\frac{r(T)}{r_0}\right)^3 = \left(\frac{r_0 - kT}{r_0}\right)^3 = 0.2.$$

$$\frac{r_0 - kT}{r_0} = \sqrt[3]{0.2} \rightarrow T = \frac{r_0}{k} (1 - \sqrt[3]{0.2}) = \frac{4 \text{ cm}}{2 \frac{\text{cm}}{\text{hrs}}} (1 - \sqrt[3]{0.2}) = 0.83 \text{ hrs} \approx 50 \text{ min}.$$

Answer: 0.83 hrs \approx 50 min.