Answer on Question #50695, Physics, Optics

3. State salient features of single slit Fraunhofer diffraction pattern. The slit is vertical and illuminated by a point source. Also, obtain an expression for intensity distribution and plot it.

Solution:

The Fraunhofer diffraction equation is used to model the diffraction of waves when the diffraction pattern is viewed at a long distance from the diffracting object.

The diffraction at a single slit of width d is shown in Figure 1. Diffraction occurs in all directions to the right of the slit.

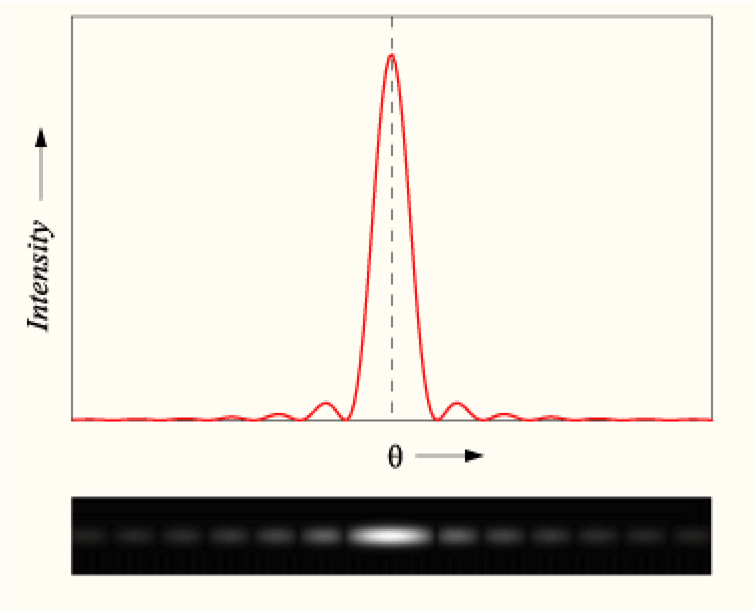


Fig.1. Graph and image of single-slit diffraction

The pattern consists of a central bright fringe (band) flanked by much weaker maxima alternating with dark fringes.

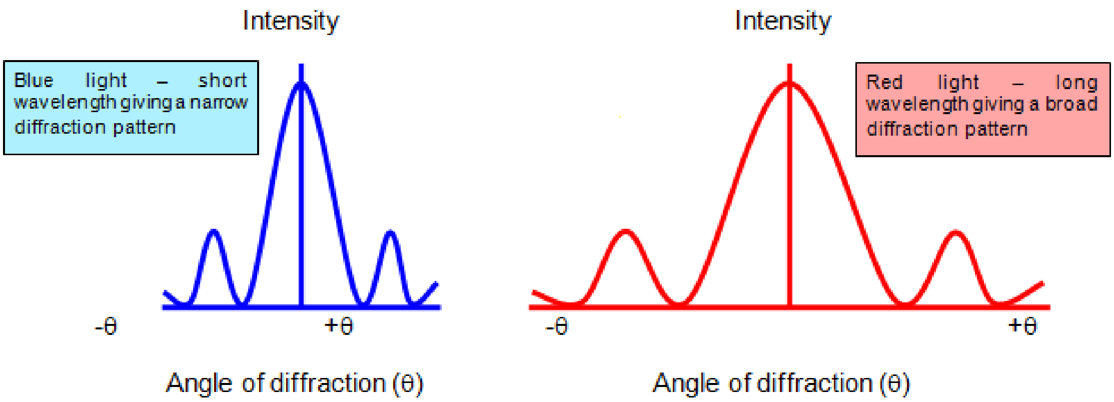
The general condition for a minimum for a single slit is:

$$m\lambda = a\sin\theta$$

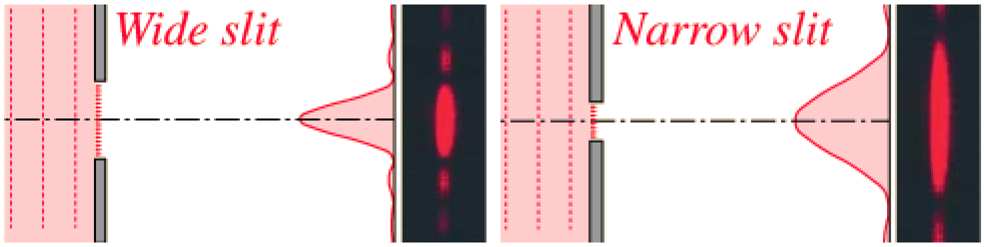
where m = 1, 2, 3, 4 and so on

- a is the width of the slit,
- θ is the angle at which the minimum intensity occurs, and
- λ is the wavelength of the light

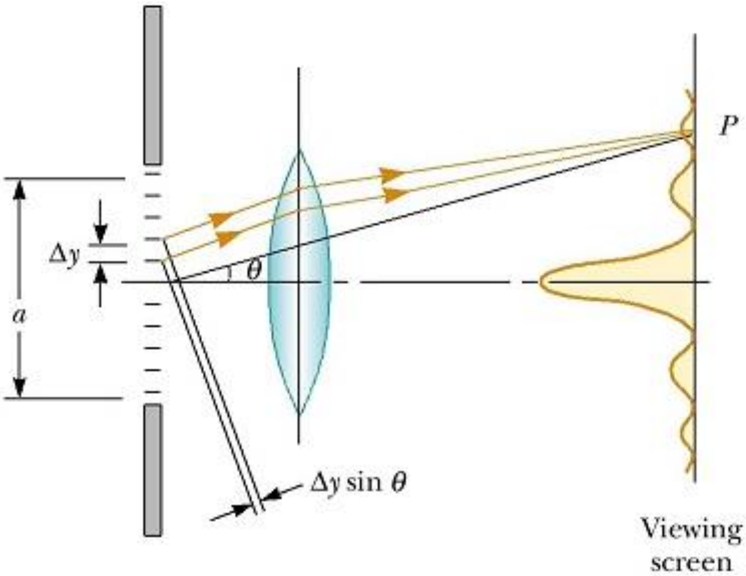
These two diagrams show the effect of a change of wavelength on the single slit diffraction pattern. The pattern for red light is broader than that for blue because of the longer wavelength of red light.



One of the characteristics of single slit diffraction is that a narrower slit will give a wider diffraction pattern as illustrated below, which seems somewhat counter-intuitive.



Intensity of single-slit diffraction patterns



We can use phasors to determine the light intensity distribution for a single-slit diffraction pattern. Imagine a slit divided into a large number of small zones, each of width Δy as shown in Figure. Each zone acts as a source of coherent radiation, and each contributes an incremental electric field of magnitude ΔE at some point P on the screen. We obtain the total electric field magnitude E at point P by summing the contributions from all the zones. The light intensity at point P is proportional to the square of the magnitude of the electric field.

The incremental electric field magnitudes between adjacent zones are out of phase with one another by an amount Δβ, where the phase difference Δβ is related to the path difference Δy sin θ between adjacent zones by the expression

$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta$$

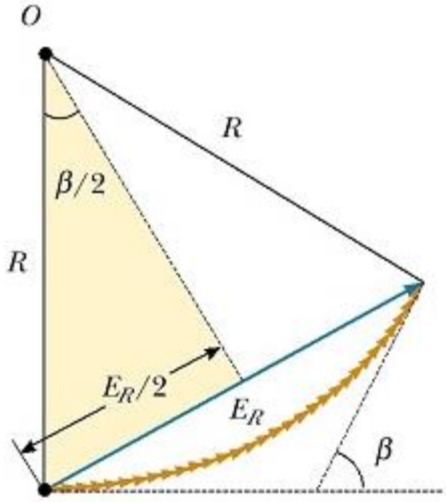
The total phase difference β between waves from the top and bottom portions of the slit is

$$\beta = N\Delta\beta = \frac{2\pi}{\lambda} N\Delta y \sin\theta = \frac{2\pi}{\lambda} a \sin\theta$$

Where a = NΔy is the width of the slit.

We can obtain the total electric field magnitude ER and light intensity I at any point P on the screen in Figure 38.7 by considering the limiting case in which Δy becomes infinitesimal (dy) and N approaches ∞.

In this limit, the phasor chains become the red curve of next Figure.



The arc length of the curve is E_0 because it is the sum of the magnitudes of the phasors (which is the total electric field magnitude at the center of the screen). From this figure, we see that at some angle θ, the resultant electric field magnitude E_R on the screen is equal to the chord length. From the triangle containing the angle β/2, we see that

$$\sin\frac{\beta}{2} = \frac{E_R/2}{R}$$

where R is the radius of curvature. But the arc length E_0 is equal to the product Rβ, where β is measured in radians. Combining this information with the previous expression gives

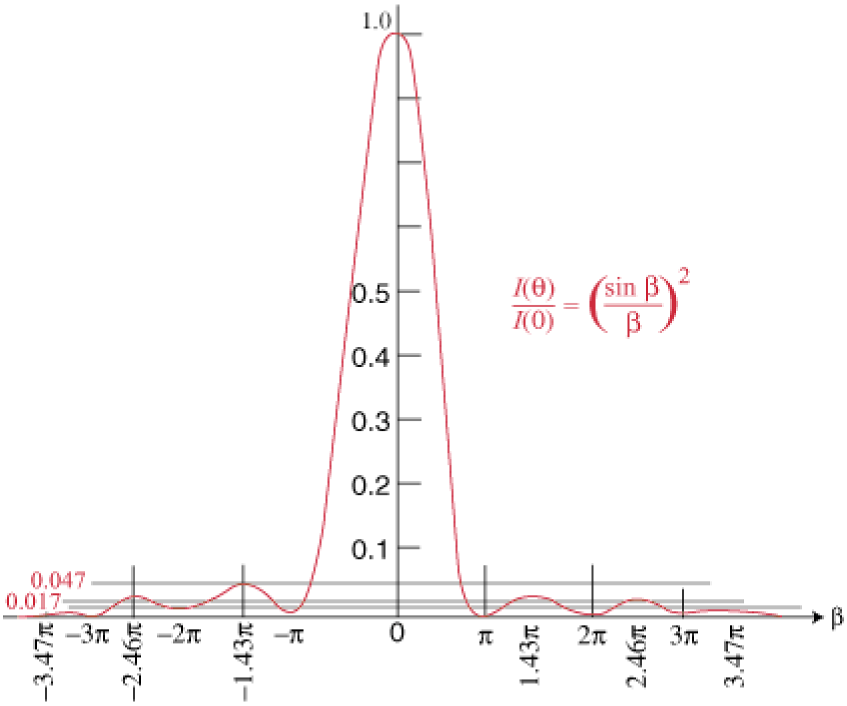
$$E_R = 2R\sin\frac{\beta}{2} = 2\left(\frac{E_0}{\beta}\right)\sin\frac{\beta}{2} = E_0\left[\frac{\sin\frac{\beta}{2}}{\beta/2}\right]$$

Because the resultant light intensity I at point P on the screen is proportional to the square of the magnitude E_R , we find that

$$I = I_{max}\left[\frac{\sin\beta/2}{\beta/2}\right]^2$$

where Imax is the intensity at θ = 0 (the central maximum). Substituting the expression for β we have

$$I = I_{max}\left[\frac{\sin(\pi a/\lambda\sin\theta)}{\pi a/\lambda\sin\theta}\right]^2$$



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