Answer on Question #50692, Physics, Optics

State salient features of single slit Fraunhofer diffraction pattern. The slit is vertical and illuminated by a point source. Also, obtain an expression for intensity distribution and plot it.

Answer

Fraunhofer diffraction deals with the limiting cases where the light approaching the diffracting object is parallel and monochromatic, and where the image plane is at a distance large compared to the size of the diffracting object.

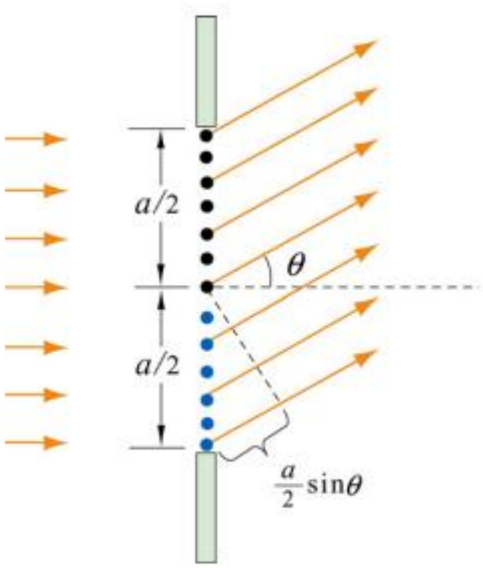

Fig.1

Let a source of monochromatic light be incident on a slit of finite width a, as shown in Fig. 1.

We will put a point source of light in the focus of a converging lens. Then after refraction in the lens parallel beams will be obtained (see Fig.2).

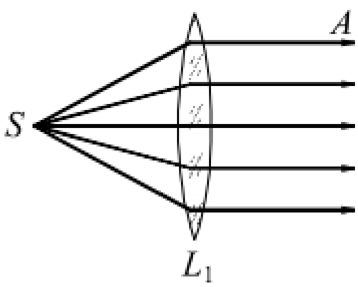

Fig.2

In diffraction of Fraunhofer type, all rays passing through the slit are approximately parallel. In addition, each portion of the slit will act as a source of light waves according to Huygens's principle. For simplicity we divide the slit into two halves. At the first minimum, each ray from the upper half will be exactly 180^{o} out of phase with a corresponding ray form the lower half. For example, suppose there are 100 point sources, with the first 50 in the lower half, and 51 to 100 in the upper half. Source 1 and source 51 are separated by a distance $a/2$ and are out of phase with a path difference $\delta = \lambda / 2$. Similar observation applies to source 2 and source 52, as well as any pair that are a distance $a/2$ apart. Thus, the condition for the first minimum is $(a/2)\sin\theta = \lambda/2$. The argument can be generalized to show that destructive interference will occur when

$$a\sin\theta = \lambda m \tag{1}$$

where $m \in \mathbb{Z}$.

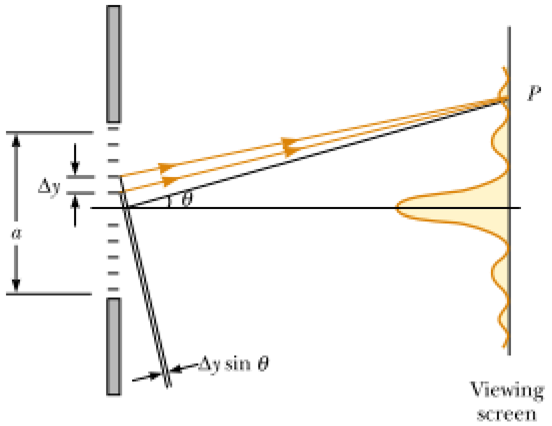

Fig.3

We can use phasors to determine the light intensity distribution for a single-slit diffraction pattern. Imagine a slit divided into a large number of small zones, each of width Δy as shown at right. Each zone acts as a source of coherent radiation, and each contributes an incremental electric field of magnitude ΔE at some point on the screen. We obtain the total electric field magnitude E at a point on the screen by summing the contributions from all the zones. The light intensity at this point is proportional to the square of the magnitude of the electric field.

The incremental electric field magnitudes between adjacent zones are out of phase with one another by an amount $\Delta\beta$, where the phase difference $\Delta\beta$ is related to the path difference $\Delta y\sin\theta$ between adjacent zones by an expression given by an argument similar to that leading to what we did with interference (Eq.(2)).

$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y\sin\theta \tag{2}$$

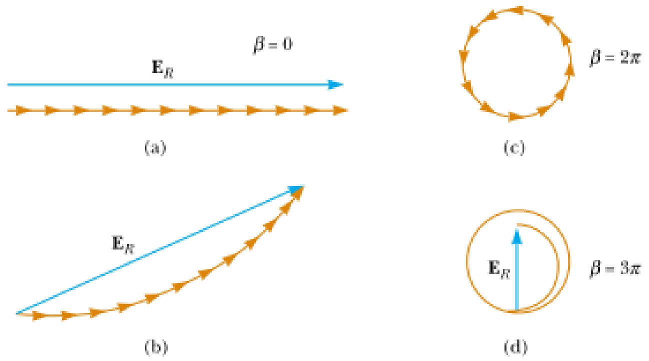

Fig.4

To find the magnitude of the total electric field on the screen at any angle θ, we sum the incremental magnitudes ΔE due to each zone. For small values of θ, we can assume that all the ΔE values are the same. It is convenient to use phasor diagrams for various angles, as shown at right. When $\theta = 0$, all phasors are aligned as in Fig.4,a because all the waves from the various zones are in phase. In this case, the total electric field at the center of the screen is $E_0 = N\Delta E$, where N is the number of zones. The resultant magnitude E_R at some small angle q is shown in Fig. 4, b, where each phasor differs in phase from an adjacent one by an amount $\Delta\beta$. In this case, E_R is the vector sum of the incremental magnitudes and hence is given by the length of the chord. Therefore, $E_R < E_0$. The total phase difference β between waves from the top and bottom portions of the slit is

$$\beta = N\Delta\beta = \frac{2\pi}{\lambda} N\Delta y \sin\theta = \frac{2\pi}{\lambda} a \sin\theta \qquad (3)$$

where $a = N \cdot \Delta y$ is the width of the slit.

As θ increases, the chain of phasors eventually forms the closed path shown in Fig.4,c. At this point, the vector sum is zero, and so $E_R = 0$, corresponding to the first minimum on the screen. Noting that $\beta = N\Delta\beta = 2\pi$ in this situation, we see from the equation derived above that $2\pi = \frac{2\pi}{\lambda} a \cdot \sin\theta_{dark} \Rightarrow \sin\theta_{dark} = \frac{\lambda}{a}$

That is, the first minimum in the diffraction pattern occurs where $\sin\theta_{dark} = \lambda / a$; At larger values of θq, the spiral chain of phasors tightens. For example, Fig. 4,d represents the situation corresponding to the second maximum, which occurs when $\beta = 360^0 + 180^0 = 540^0$. The second minimum (two complete circles, not shown) corresponds to $\beta = 720^0$, which satisfies the condition $\sin\theta_{dark} = \frac{2\lambda}{a}$.

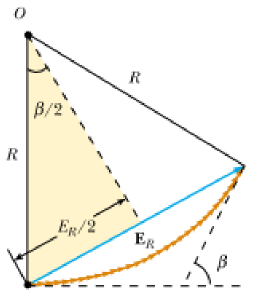

Fig.5

We can obtain the total electric-field magnitude E_R and light intensity I at any point on the screen by considering the limiting case in which Δy becomes infinitesimal (dy) and N approaches infinity. In this limit, the phasor chains shown previously become the curve shown at right. The arc length of the curve is E_0 because it is the sum of the magnitudes of the phasors (which is the total electric field magnitude at the center of the screen). From Fig.5 , we see that at some angle β, the resultant electric field magnitude E_R on the screen is equal to the chord length. From the triangle containing the angle $\beta/2$, we see that

$$\sin\frac{\beta}{2} = \frac{E_R/2}{R} \tag{4}$$

where R is the radius of curvature. But the arc length E_0 is equal to the product $R\beta$, where is measured in radians. Combine this information with the previous expression to write an expression for E_R as a function of E_0 and β

$$E_R = 2R\sin\frac{\beta}{2} = 2\left(\frac{E_0}{\beta}\right)\sin\frac{\beta}{2} = E_0\left[\frac{\sin(\beta/2)}{\beta/2}\right] \tag{5}$$

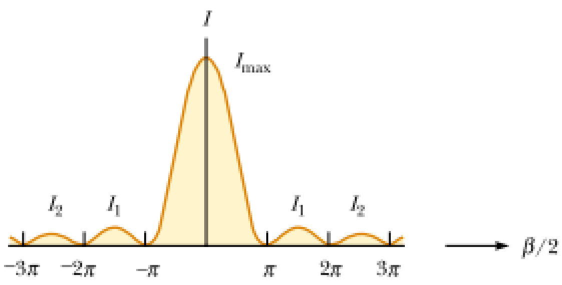

Fig.6

Because the resultant light intensity I (see Fig.6) at a point on the screen is proportional to the square of the magnitude E_R , we find that

$$I = I_{\max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 \tag{6}$$