Answer on Question #50691-Physics-Optics

Obtain the expression for shift in fringes when a thin transparent sheet is introduced in the path of one of the waves in a double slit interference experiment.

Solution

Let us now investigate the change in the interference pattern produced by introducing a thin transparent plate in the path one of the two interference beams as shown in figure. Let a thin transparent sheet of thickness t and refractive index μ be introduced in the path of wave from one slit S_1. It is seen from the figure that light reaching the point P from source S_1 has to traverse a distance t in the sheet and a distance $(S_1P - t)$ in the air. If c and v are velocities of light in air and in transparent sheet respectively, then the time taken by light to reach from S_1 to P is given by

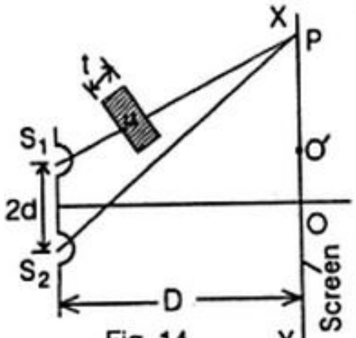

Fig. 14

$$= \frac{(S_1P - t)}{c} + \frac{t}{v} = \frac{(S_1P - t)}{c} + \frac{\mu t}{c} = \frac{1}{c}[(S_1P - t) + \mu t]$$
$$= \frac{1}{c}(S_1P + (\mu - 1)t).$$

Thus by introducing thin plate the effective optical path in air is increased by an amount $(\mu - 1)t$. In the absence of the sheet, let O be the position of the central bright fringe, which corresponds to equal optical path from S_1 to S_2. In the presence of the sheet, the two optical paths S_1O and S_2O become unequal and therefore the two waves from S_1 and S_2 don't arrive at O simultaneously, therefore the central fringe is shifted to a point O' such that at O' two optical paths become equal. Such a shift results for all fringes.

Now the effective path difference at any point P on the screen

$$\Delta' = S_2P - (S_1P + (\mu - 1)t) = S_2P - S_1P - (\mu - 1)t.$$

If the distance between two sources S_1 and S_2 be $2d$, the distance of the screen from the sources be D and the position of the n^{th} bright fringe is x_n, then the path difference in absence of the sheet is

$$S_2P - S_1P = \frac{2d}{D}x_n.$$

Thus the effective path difference

$$\Delta' = \frac{2d}{D}x_n - (\mu - 1)t.$$

If the point P is now at the center of n^{th} bright fringe, then from the condition of bright fringe

$\Delta' = \frac{2n\lambda}{2}, where\ n = 0,1,2, \dots$ we have

$$\frac{2d}{D}x_n - (\mu - 1)t = n\lambda \rightarrow x_n = \frac{D}{2d}[n\lambda + (\mu - 1)t].$$

In the absence of the sheet $t = 0$, the distance of n^{th} bright fringe from O is $\frac{D}{2d}[n\lambda]$. Therefore, the displacement of n^{th} bright fringe is

$$x_0 = \frac{D}{2d}[n\lambda + (\mu - 1)t] - \frac{D}{2d}[n\lambda] = \frac{D}{2d}(\mu - 1)t.$$

It is clear that x_0 is independent of n, that is the displacement is same for all bright fringes. Similar expression can also be obtained for dark fringes. Thus the introduction of the transparent sheet in the path of one of the waves simply displaces the entire fringe system through a distance $\frac{D}{2d}(\mu - 1)t$ towards the side on which the sheet is placed.